Control Charts

An Introduction to Statistical Process Control

Business Performance Improvement

Course Content

- Prerequisites
- Course Objectives
- What is SPC?
- Control Chart Basics
- Out of Control Conditions
- SPC vs. SQC
- Individuals and Moving Range Chart
- Central Limit Theorem
- X-bar and Range Charts
- Advanced Control Charts
- Attribute Charts
- Final Points
- Reference Section

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Prerequisites

- Learners should be familiar with the following concepts prior to taking this course
  - Variation
  - Mean and Standard Deviation
  - Histograms
  - Normal Distributions
  - Cp and Cpk
- Capability Course is available on BPI website if you need to review these topics

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- Upon completion of this course, participants should be able to:
  - Understand the basics of creating variable and attribute control charts
  - Understand the concepts of advanced control charting
  - Identify an out of control condition
  - Identify which control chart to use with each process
  - Calculate control limits for any control chart

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Statistical Process Control (SPC)

- SPC is the application of **statistical methods** to identify and **control** the special cause of variation in a **process**
- SPC is a preventative tool to:
  - Assess the consistency of a process
  - Monitor a process to determine when it has changed
  - Reduce variation in a process

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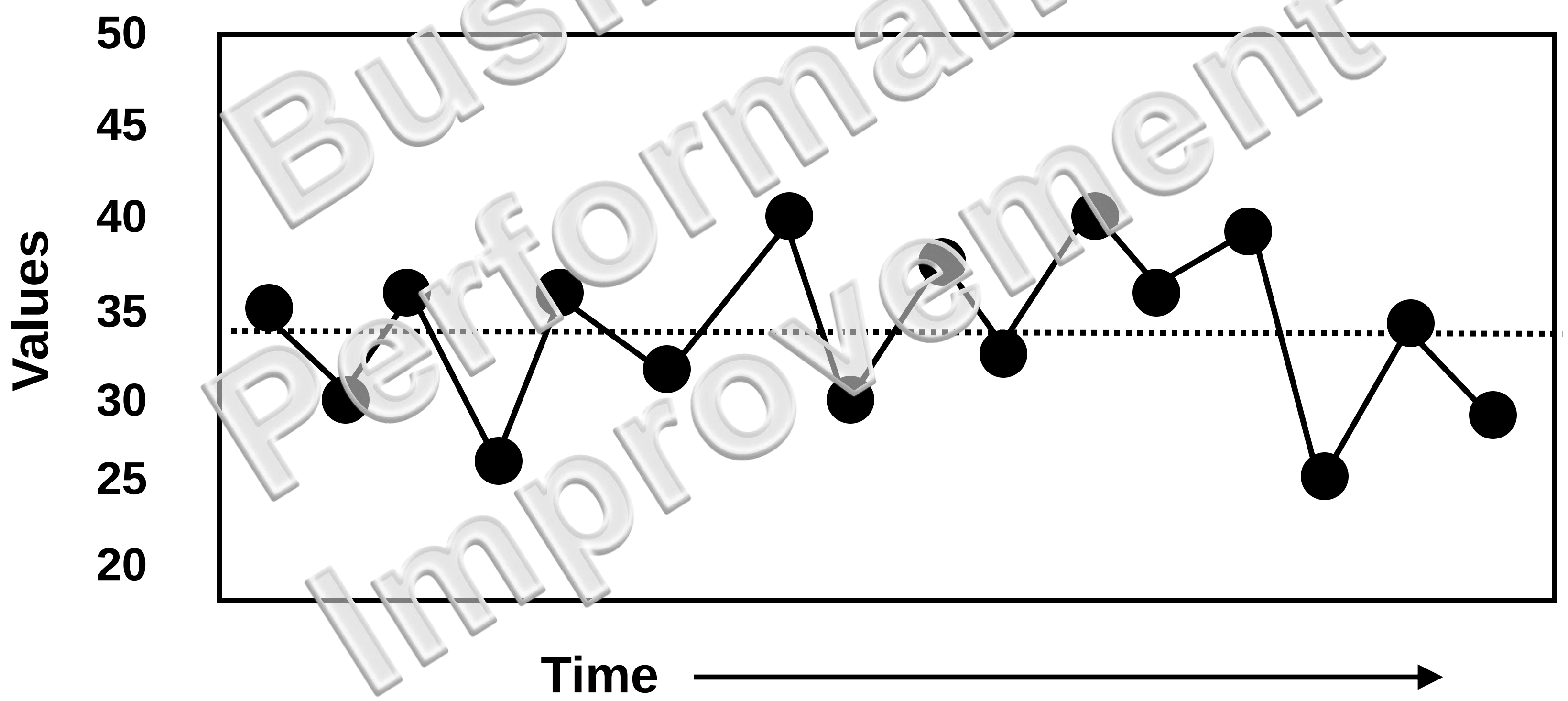
- Graph that displays observed data in a time sequence



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- Run chart with calculated control limits
  - 3 standard deviations above and below average
- Help distinguish process variation due to assignable or **"common"** causes from those due to unassignable or **"special"** causes
- Used to detect whether a process is statistically stable

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History

- Invented by Walter A. Shewhart while working for Bell Labs in the 1920s
- W. Edwards Deming became the foremost champion of Shewhart's work
  - long career as an industrial consultant in Japan, spread use of the control charts throughout the 1940s and 1950s

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Special vs. Common Cause

- Special Cause Variation
  - Data points outside of control limits
  - Trend or shift pattern within limits
  - Can be removed from a process
- Common Cause Variation
  - Noise within the system, typical, expected
  - Data points randomly occurring within the control limits
  - Always exists in a process, but can be reduced

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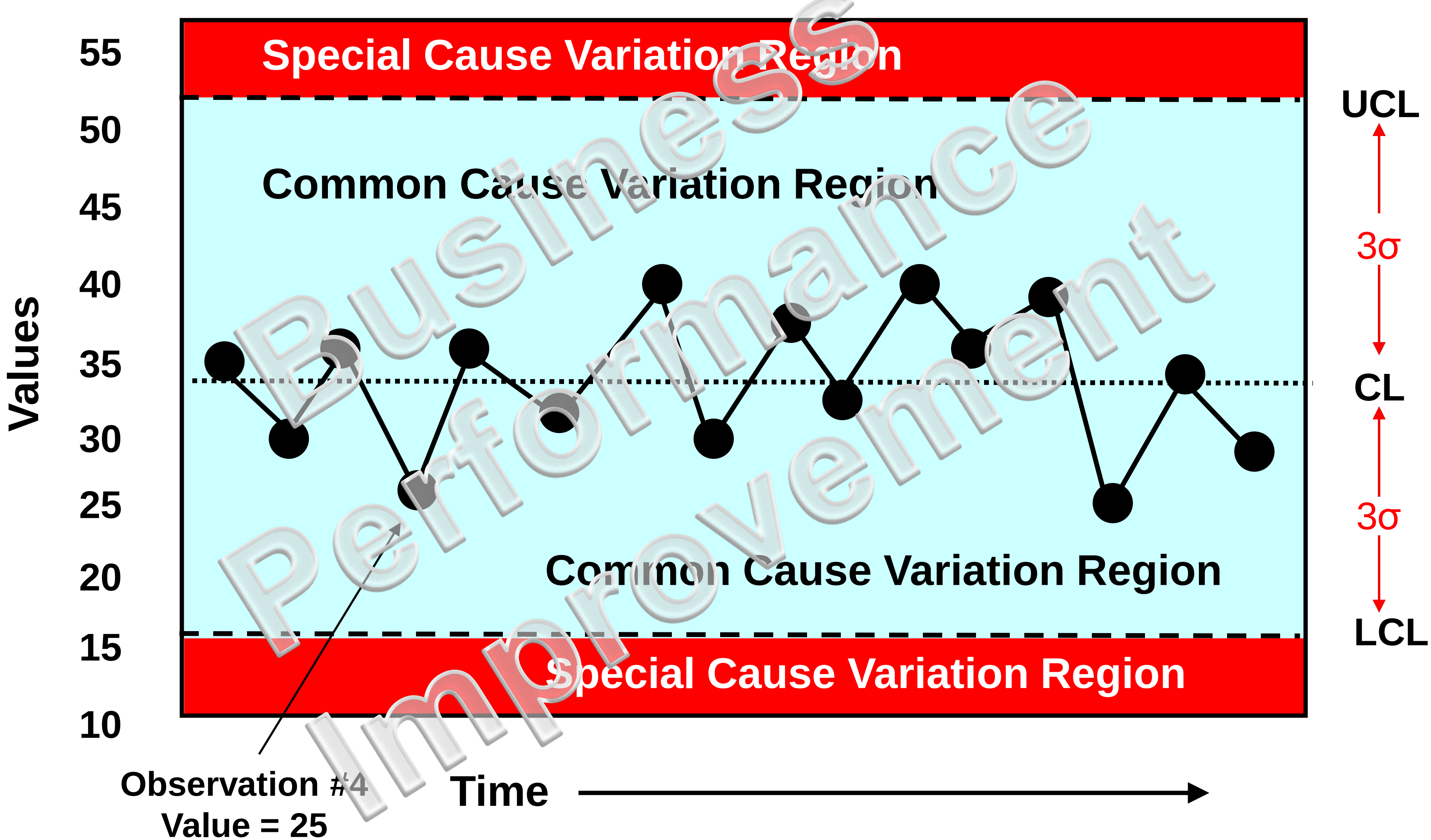
Control Chart Basics



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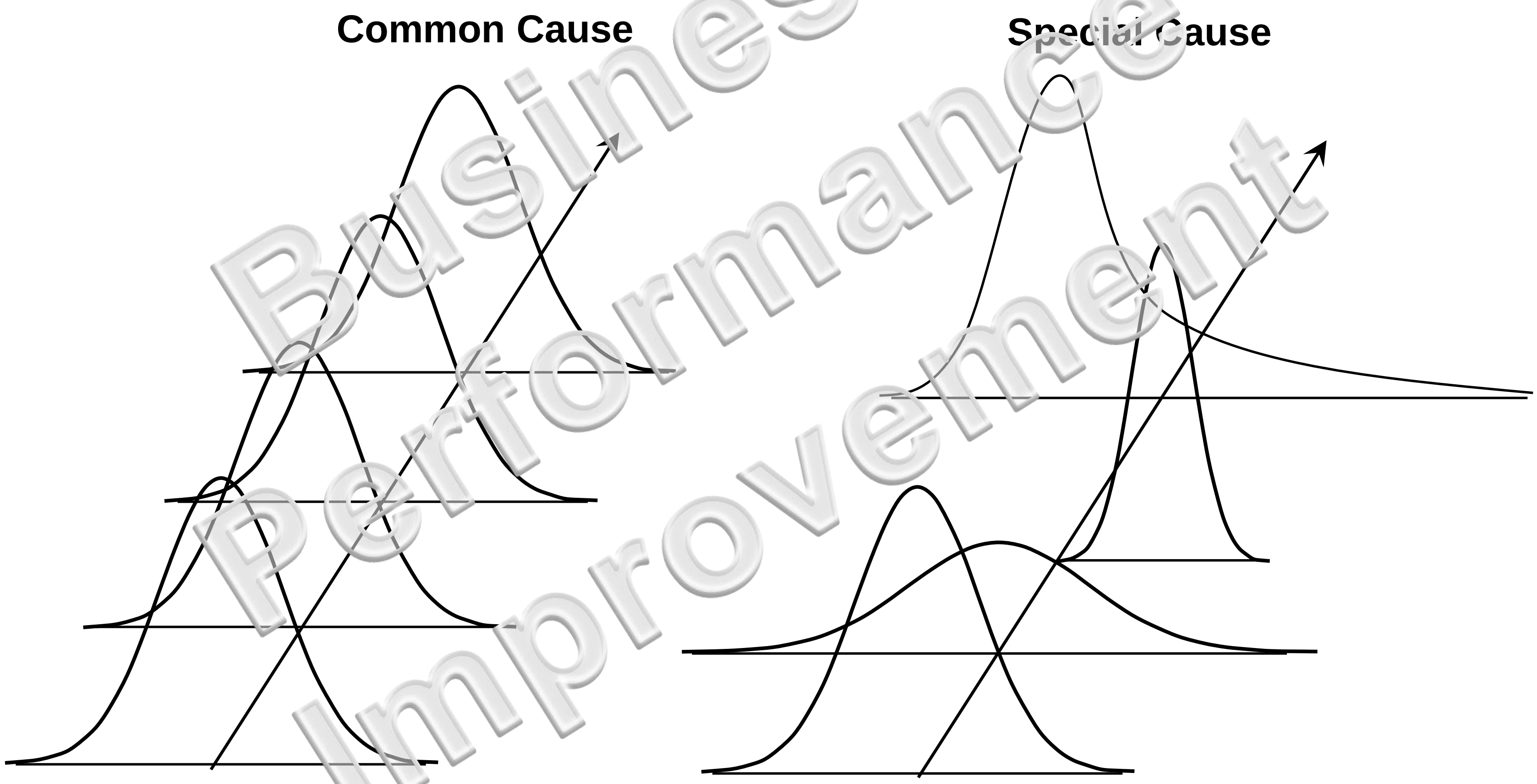
Special vs. Common Cause



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- Process: Driving to Work
- Average Time: 12 minutes
- Standard Deviation: 2.5 minutes

- Common Causes
  - Wind speed, miss one green light, driving speed, number of cars on road, time when leaving house, rainy weather
- Special Causes
  - Stop for school bus crossing, traffic accident, pulled over for speeding, poor weather conditions, car mechanical problems, construction detour, stoplights not working properly, train crossing

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Control Limits

- **Centerline** (CL) = average value of observations
- **Upper Control Limit** (UCL) ~ 3 standard deviations ABOVE the centerline
- **Lower Control Limit** (LCL) ~ 3 standard deviation BELOW the centerline
- Control limits are set when process is "in control" or "stable"
  - Fixed at baseline value
  - Adjusted for improvements
  - Never widened
- Control limits are **not** specification limits

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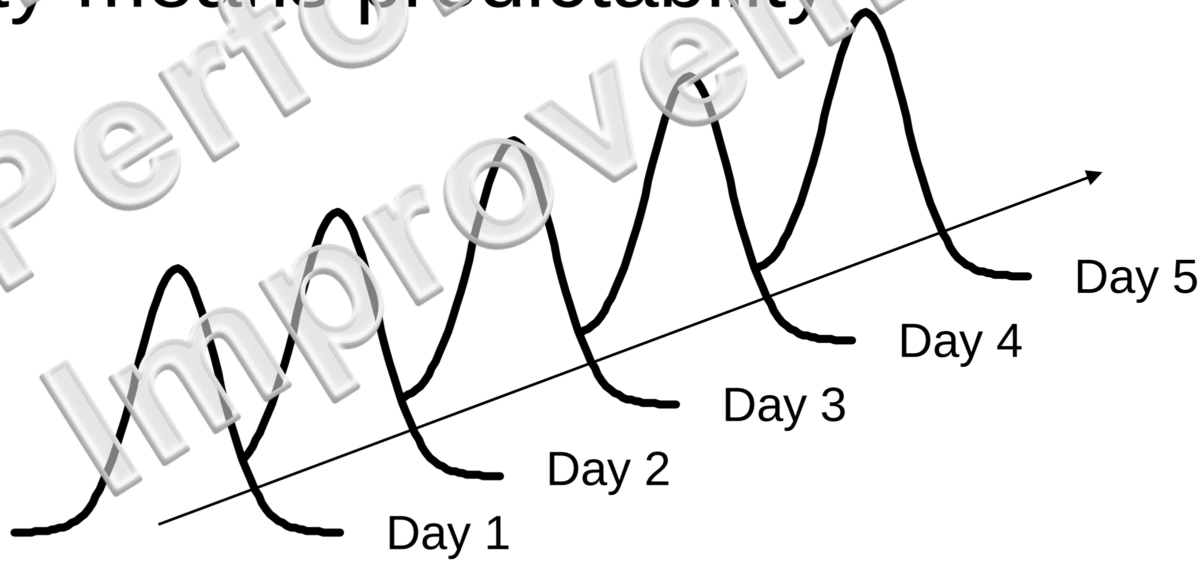
- A process is considered “in control” or “stable” when the data does not show any out of control conditions on the control chart
- Stability means predictability



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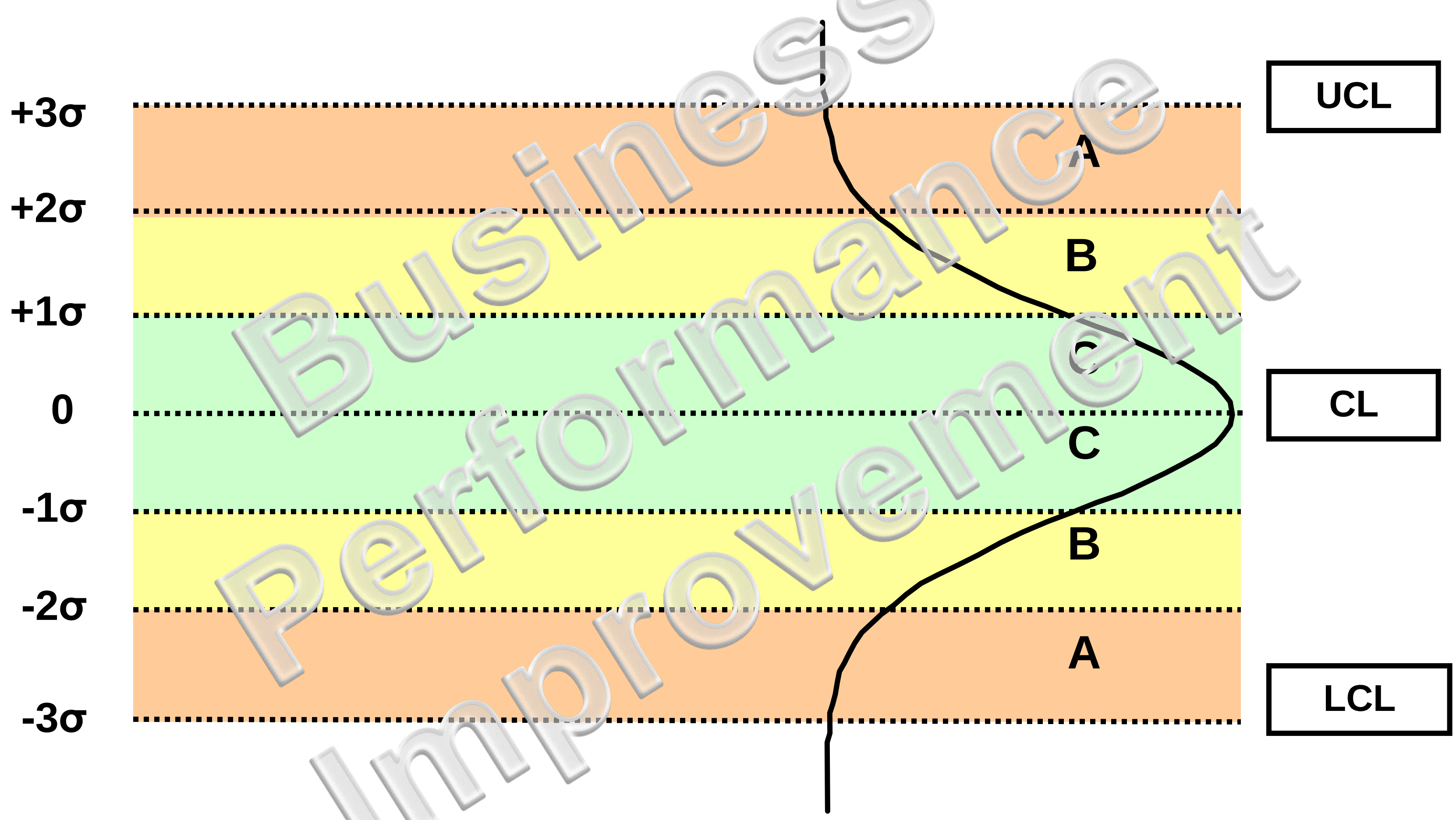
Control Limit Zones



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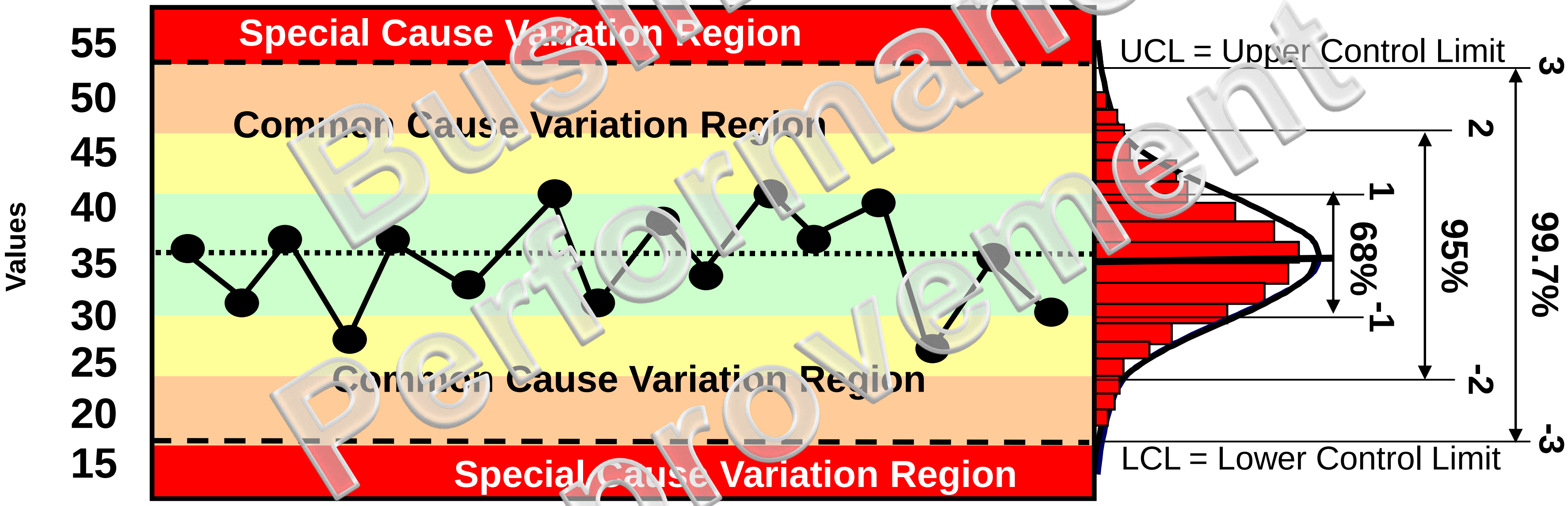
Histogram vs. Control Chart



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Out of Control (OOC) Conditions

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Out of Control Conditions

- Outside control limits is easiest to identify
- Patterns highlight other out of control conditions
  - Trends (increasing/decreasing points)
  - Shifts (data jumps higher or lower than normal)
  - Inconsistencies (not random, more or less variation than history)

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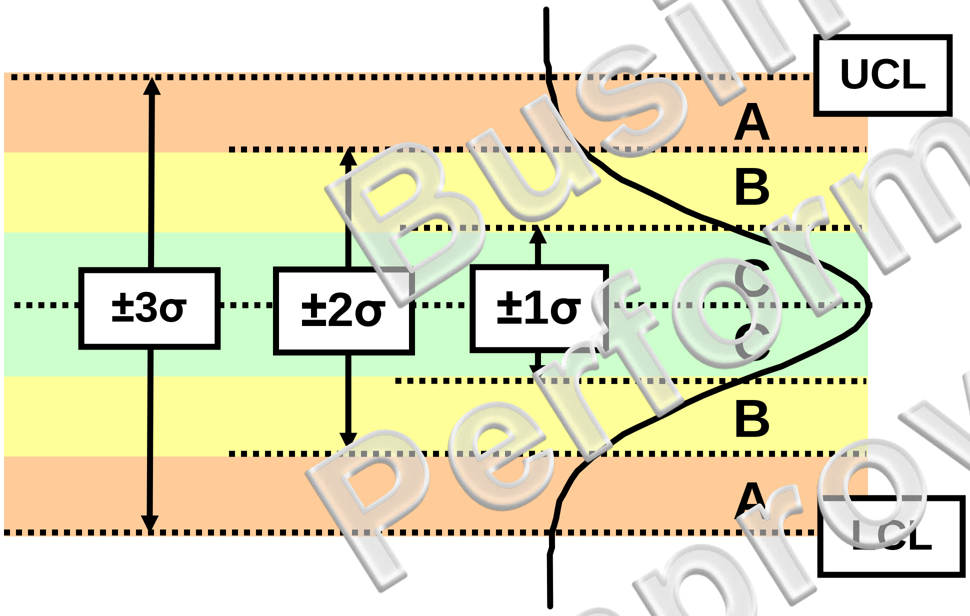
Western Electric Rules for Control



- ✓Any point outside control limits
- ✓7 consecutive points on same side of centerline
- ✓7 consecutive points increasing or decreasing
- ✓2 of 3 points in same zone A or beyond
- ✓4 of 5 points in same zone B or beyond
- ✓14 consecutive points alternating up and down
- ✓14 consecutive points in either zone C

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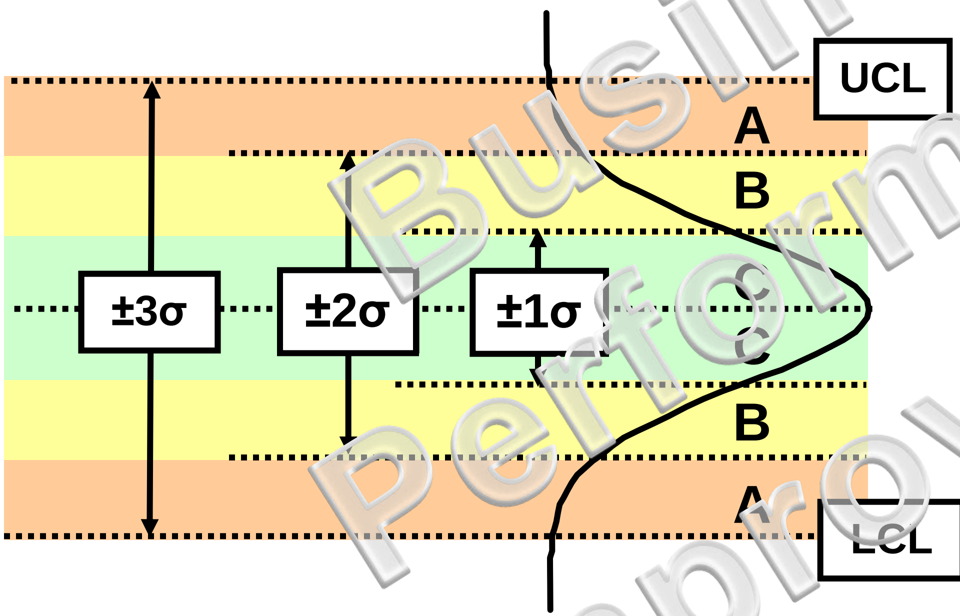
Nelson Tests for Control



- ✓ Any point outside control limits
- ✓ **9 consecutive points on same side of centerline**
- ✓ **6 consecutive points increasing or decreasing**
- ✓ 2 of 3 points in same zone A or beyond
- ✓ 4 of 5 points in same zone B or beyond
- ✓ 14 consecutive points alternating up and down
- ✓ **15 consecutive points in either zone C**
- ✓ **8 points in a row outside zone C, same side of centerline**

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False Alarm Rates are the Key

Nelson	False Alarm Rate
Any point outside control limits	.0027
9 consecutive points on same side of centerline	Approx .003
6 consecutive points increasing or decreasing	Approx .003
2 of 3 points in same zone A or beyond	.00305
4 of 5 points in same zone B or beyond	.0043
14 consecutive points alternating up and down	Approx .004
15 consecutive points in either zone C	Approx .003
8 points in a row outside zone C, either side of centerline	Approx .003

The Nelson tests are designed so that the false alarm rates for all tests are approximately the same. The Western Electric rules do not have this property.

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9 Consecutive on Same Side

- If process is stable and normal, 50% of data will be above centerline, 50% will be below centerline
- Probability of getting 9 straight on one side, same as flipping coin and getting 9 straight heads

$(.5)^{10}$ = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = .001

$(.5)^{9}$ = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = .002

$(.5)^{8}$ = 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5 = .004

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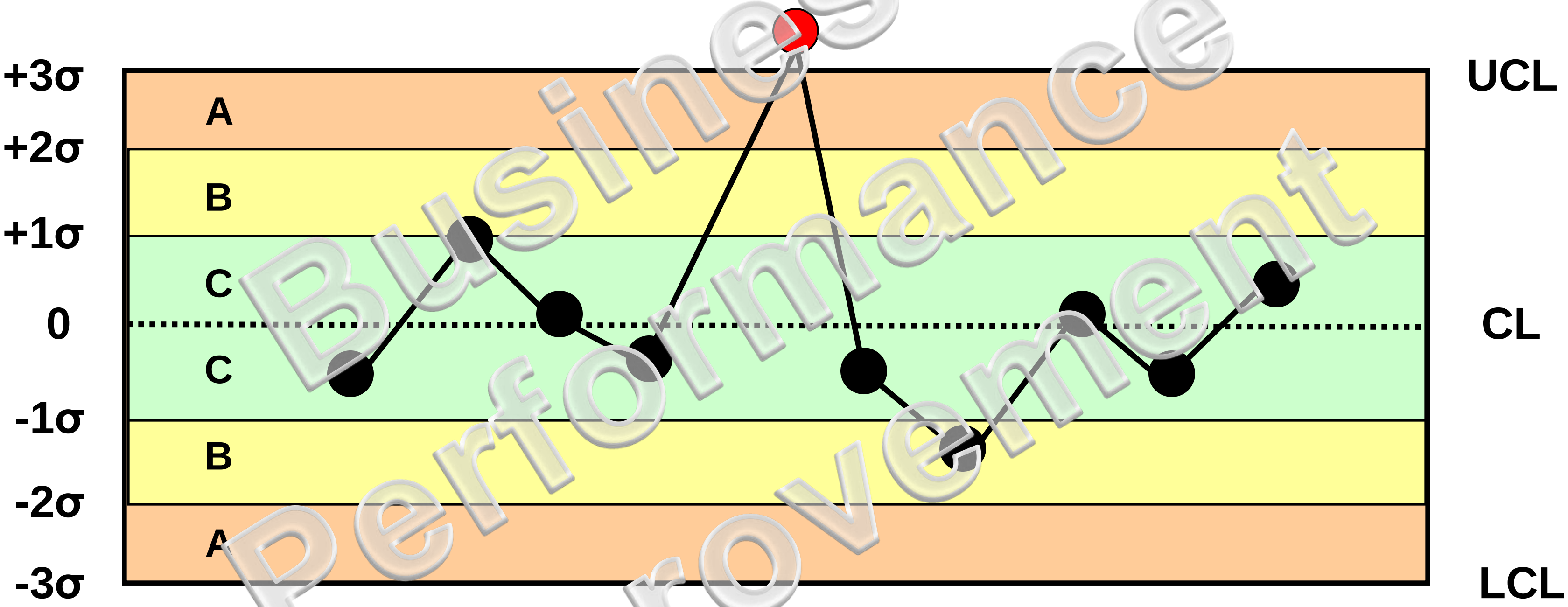
Nelson Test #1



Rule 1: Any point outside control limits

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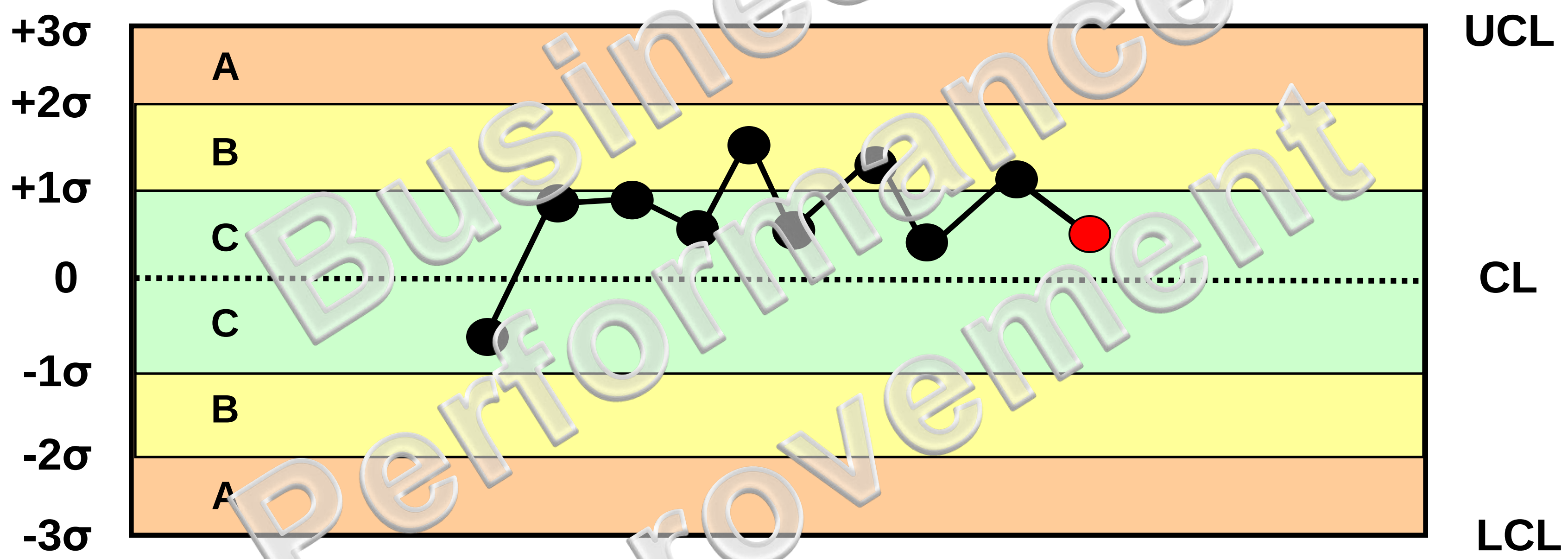
Nelson Test #2



Rule 2: 9 consecutive points on same side of centerline

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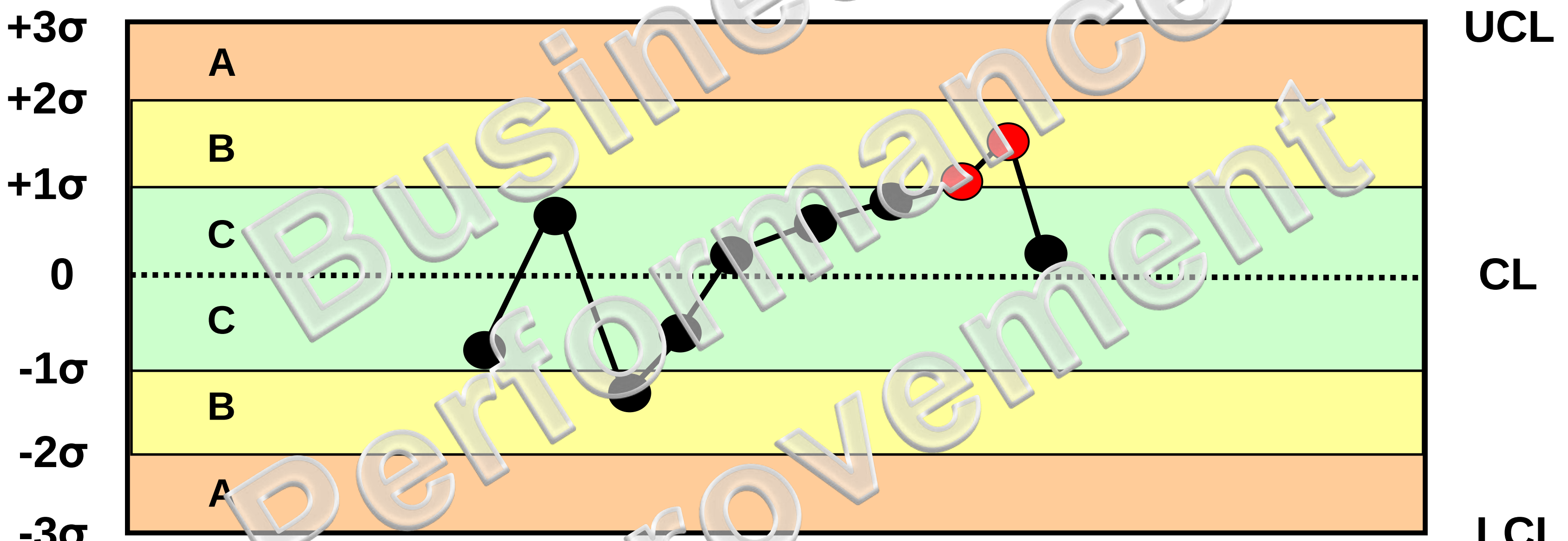
Nelson Test #3



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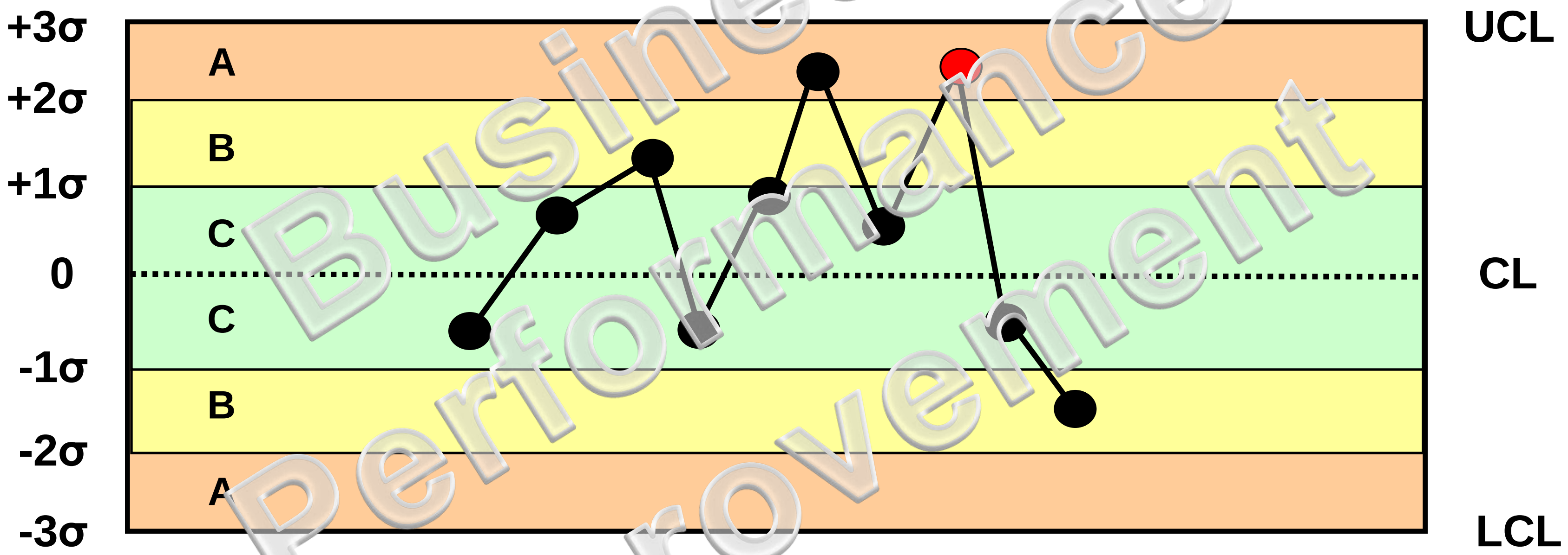
Rule 3: 6 consecutive points increasing or decreasing

Nelson Test #4



Rule 4: 2 of 3 points in same zone A or beyond

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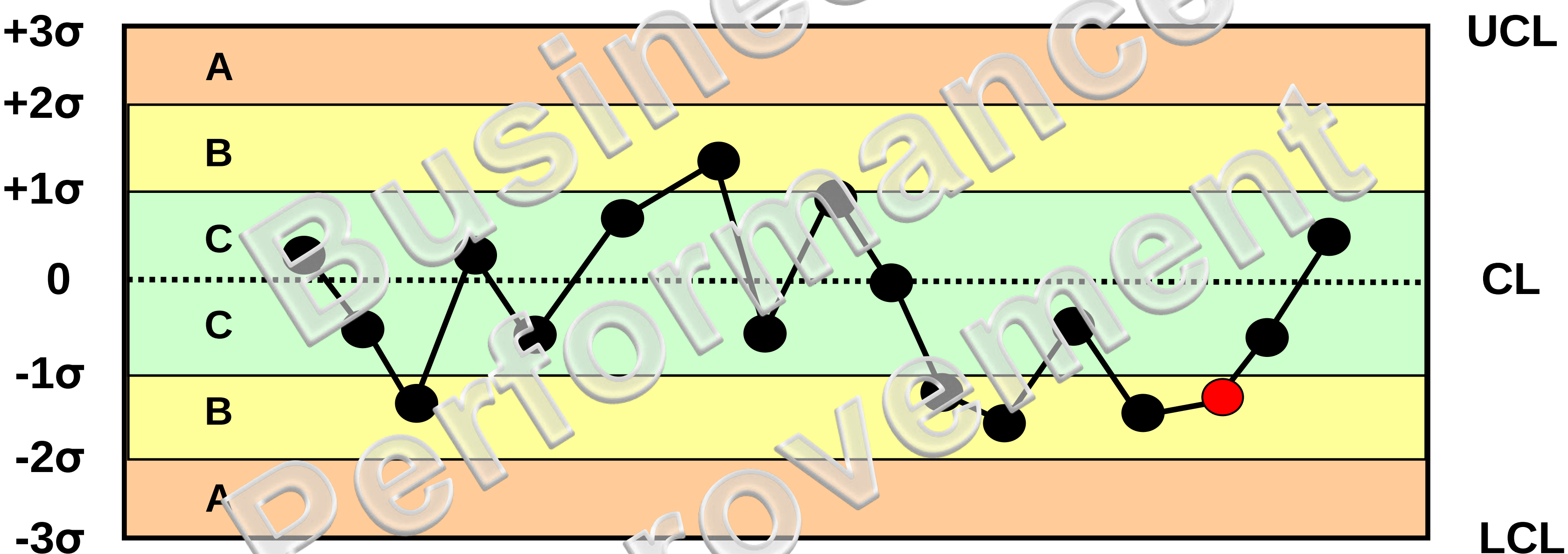
Nelson Test #5



Rule 5: 4 of 5 points in same zone B or beyond

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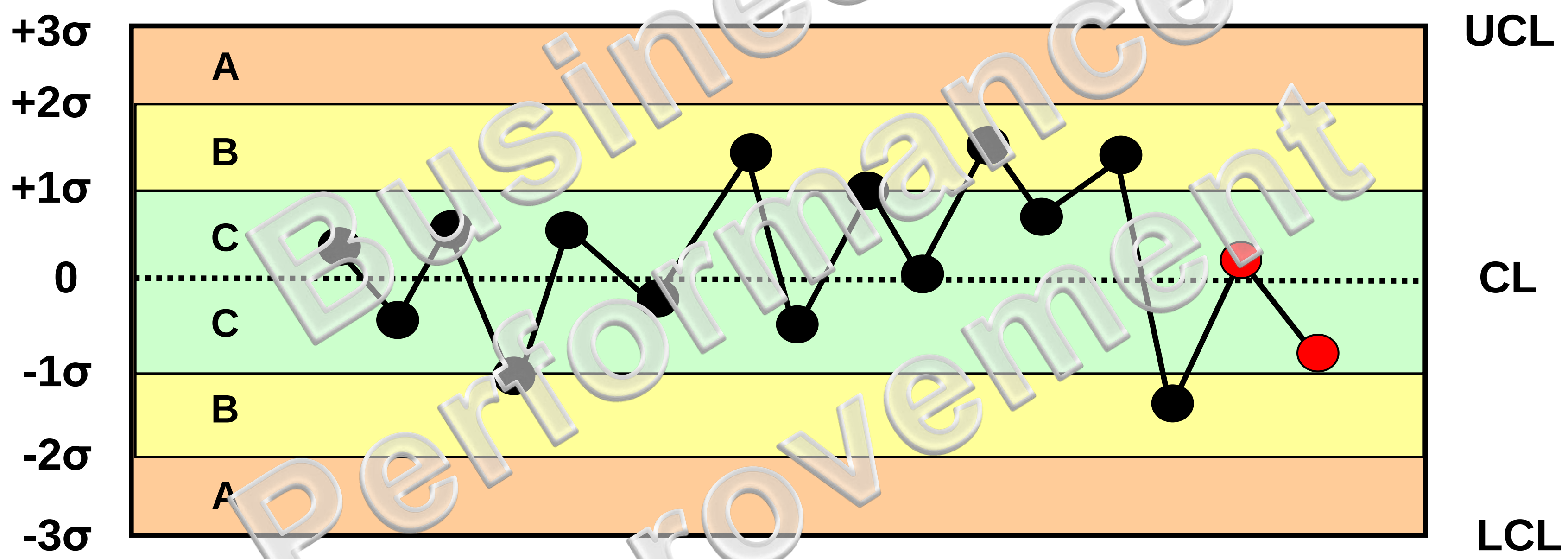
Nelson Test #6



Rule 6: 14 consecutive points alternating up and down

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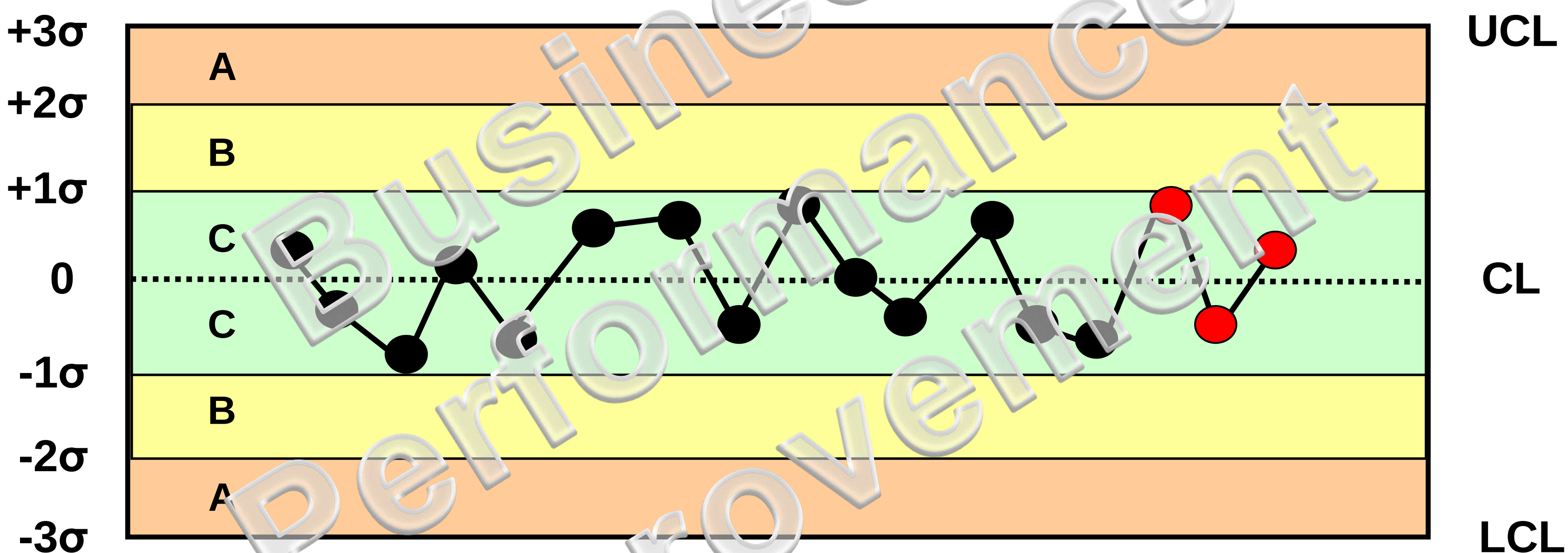
Nelson Test #7



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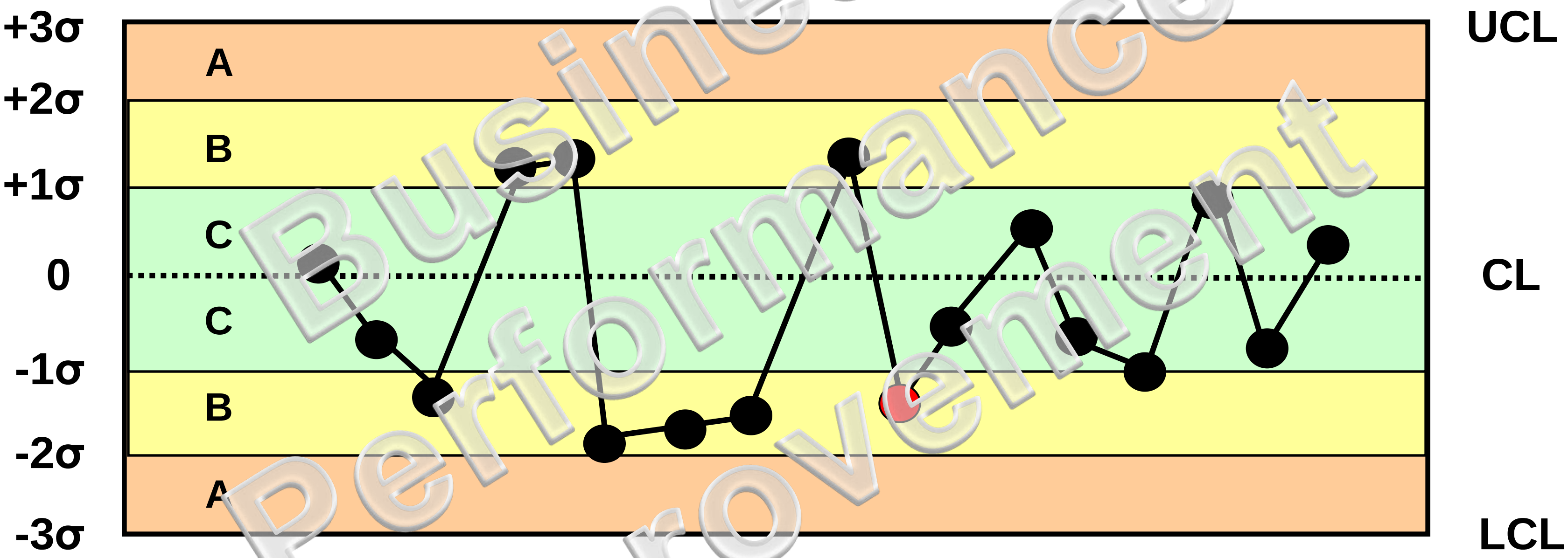
Rule 7: 15 consecutive points in either zone C

Nelson Test #8



Rule 8: 8 points in a row outside zone C, either side

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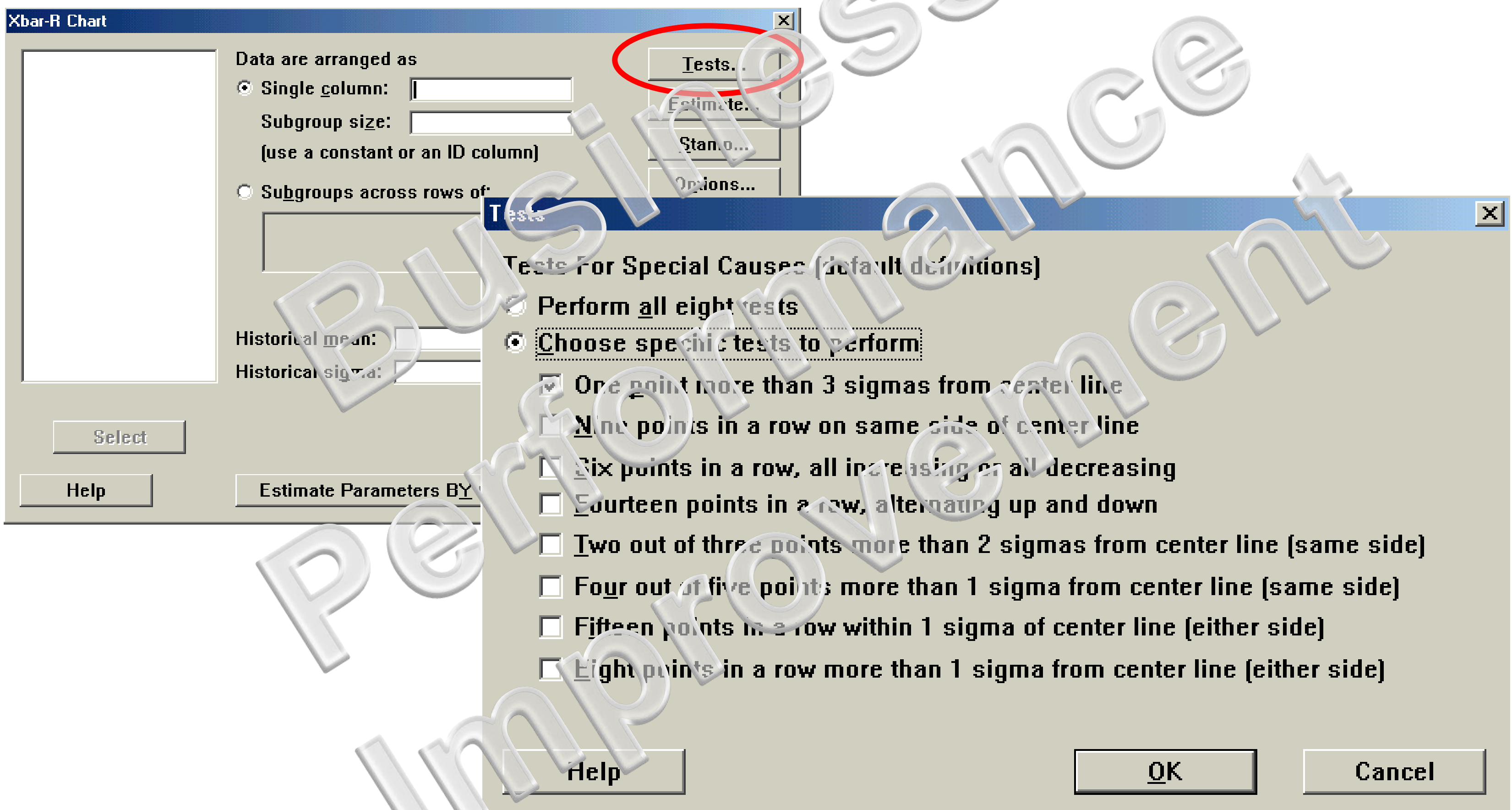
Stat > Control Charts > Xbar-R



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SPC vs. SQC

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Evolution of control charts

SPC

Statistical Process Control

SQC

Statistical Quality Control

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Individuals Chart

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Individuals and Moving Range Chart

- Most common type of control chart
  - Each individual value plotted over time
  - Difference from previous value to current value plotted on Moving Range chart
  - Moving Range average used to calculate control limits for individual readings

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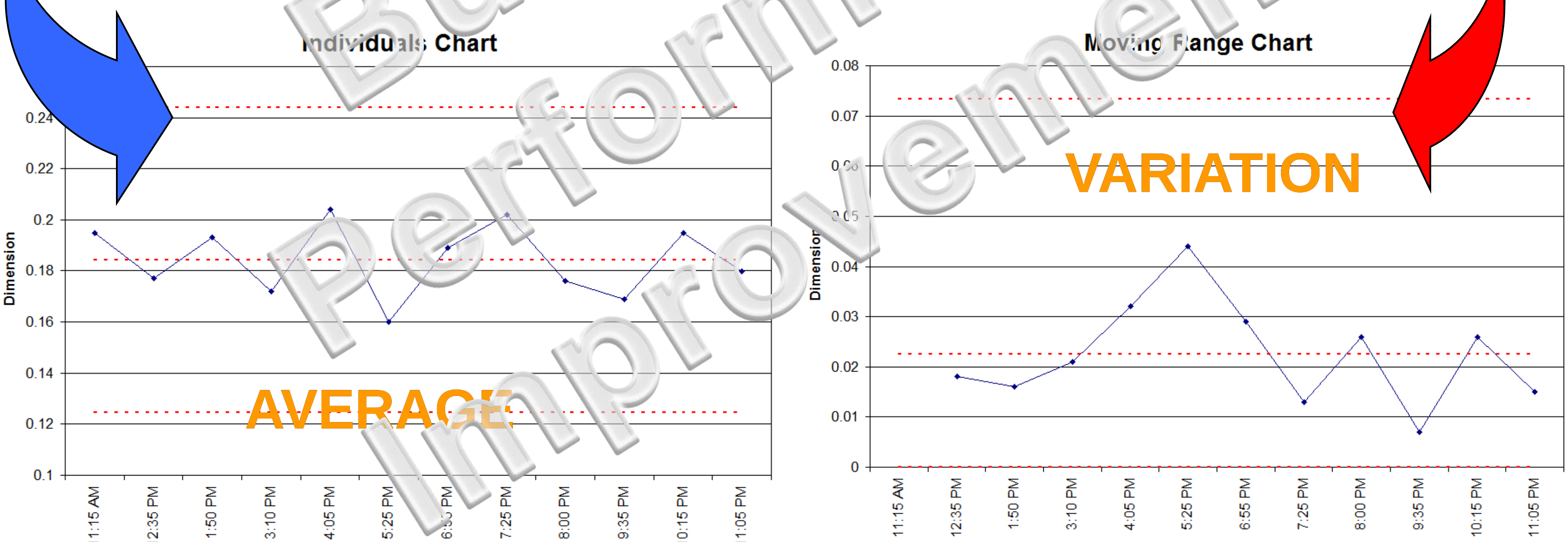
Individuals and MR example

Subgroups	11:15 AM	12:35 PM	1:50 PM	3:10 PM	4:05 PM
SAMPLE	0.195	0.177	0.193	0.172	0.204
MOVING RANGE		0.018	0.016	0.021	0.032



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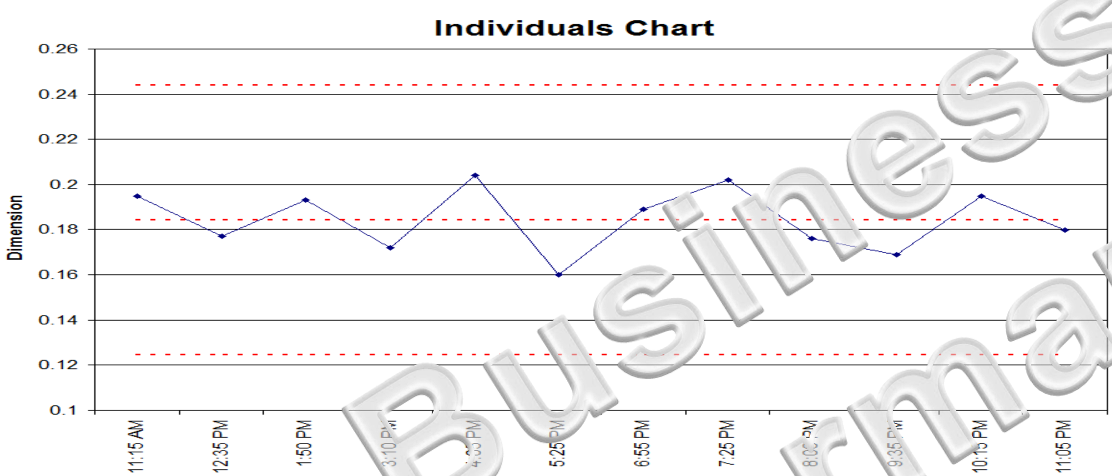
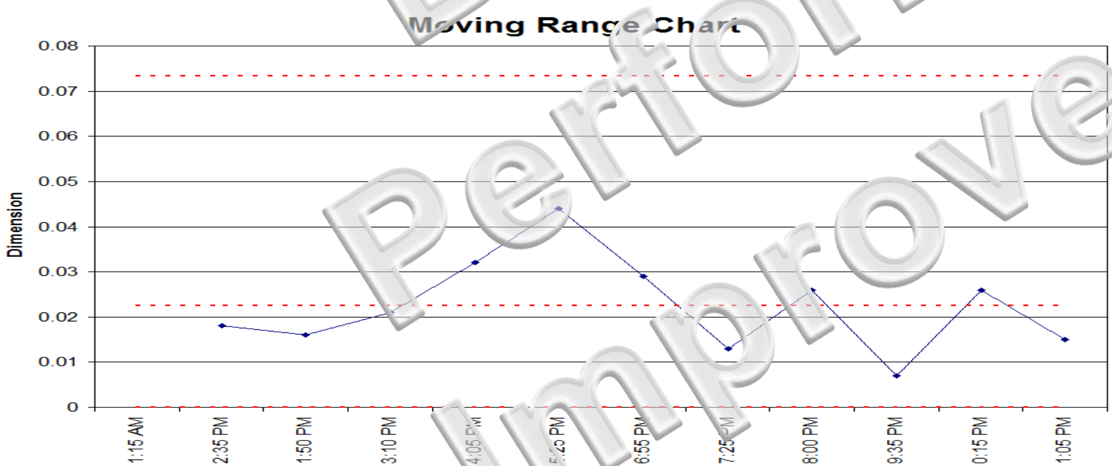
Individuals and MR UCL and LCL



$$UCL_{\overline{X}} = \overline{X} + E_2 \overline{R}$$

$$LCL_{\overline{X}} = \overline{X} - E_2 \overline{R}$$



$$UCL_{MR} = D_4 \overline{R}$$

$$LCL_{MR} = D_3 \overline{R}$$

E_2, D_3 and D_4 are constants in table at end of section

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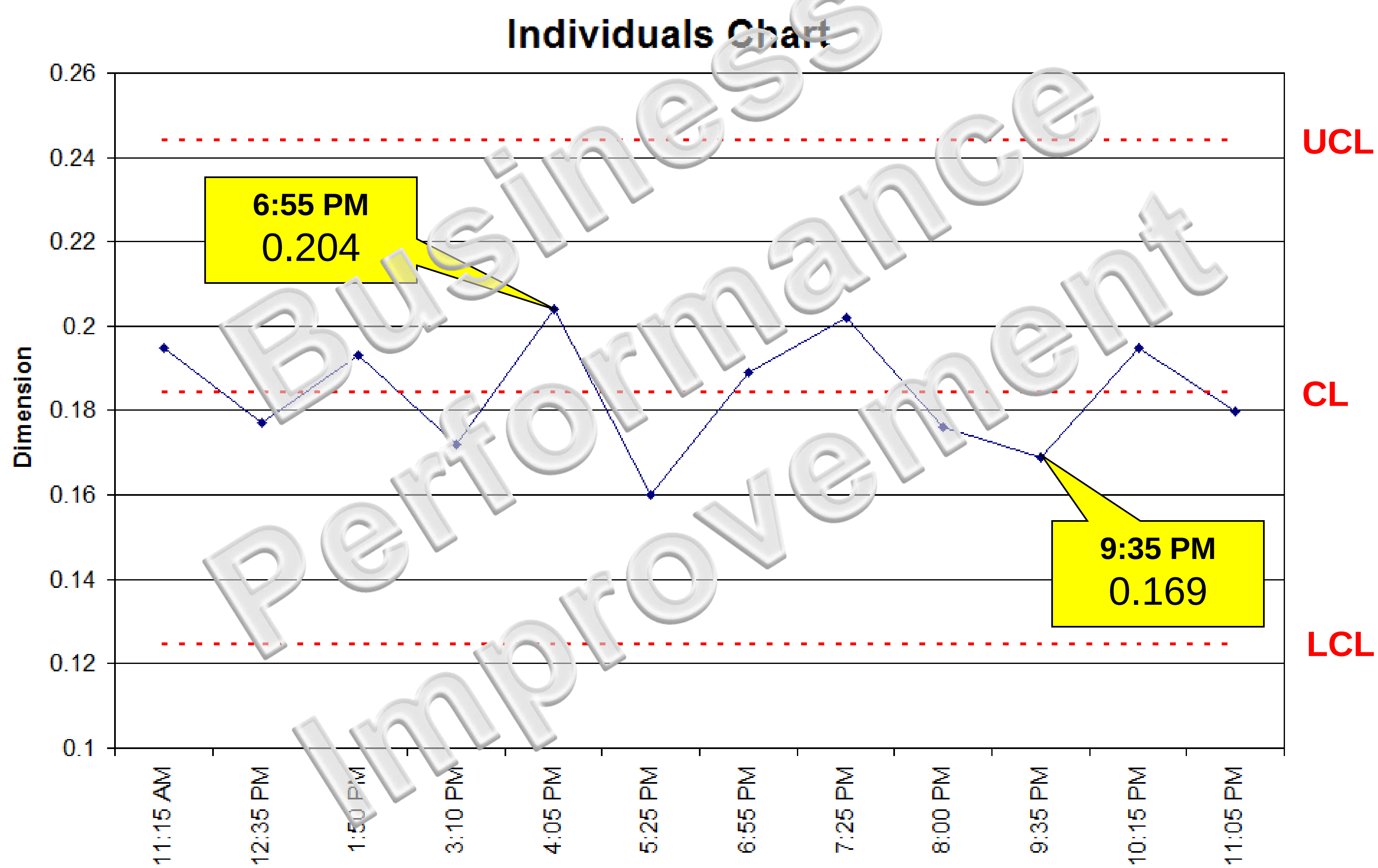
Individuals Chart



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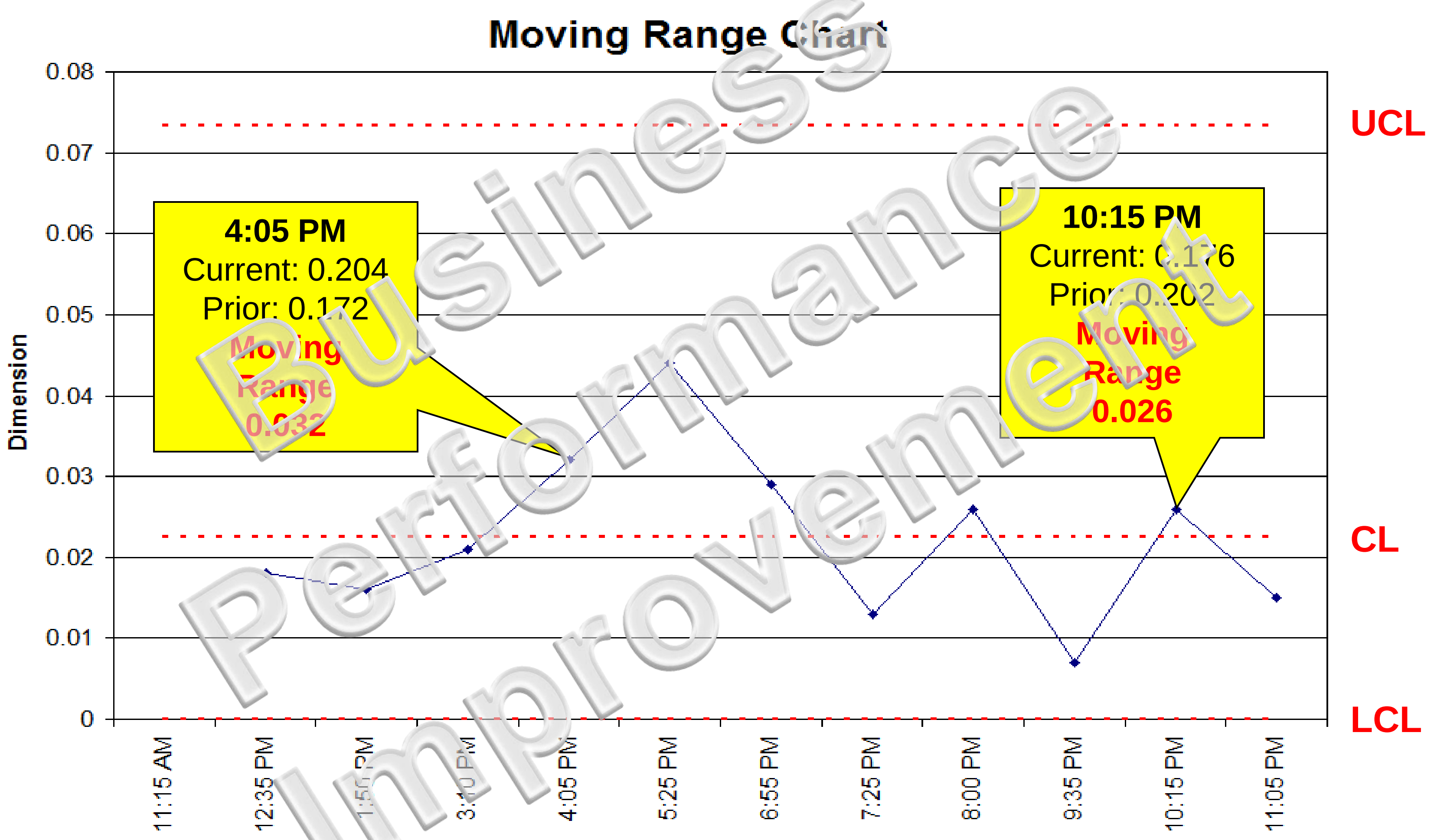
Moving Range Chart



Moving Range = Current Data Value – Prior Data Value

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Non-Normal Data

- Individuals control charts rely on assumption that data is normally-distributed
- If data does not pass normality test, what can be done?
  - Transform the data (difficult)
  - Use central limit theorem to normalize the data
    - X-bar and R chart does this for you

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Central Limit Theorem

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Central Limit Theorem

- No matter what the shape of the original distribution (non-normal), the **sampling** distribution of the mean approaches a normal distribution
  - normal distribution is approached very quickly as subgroup size increases

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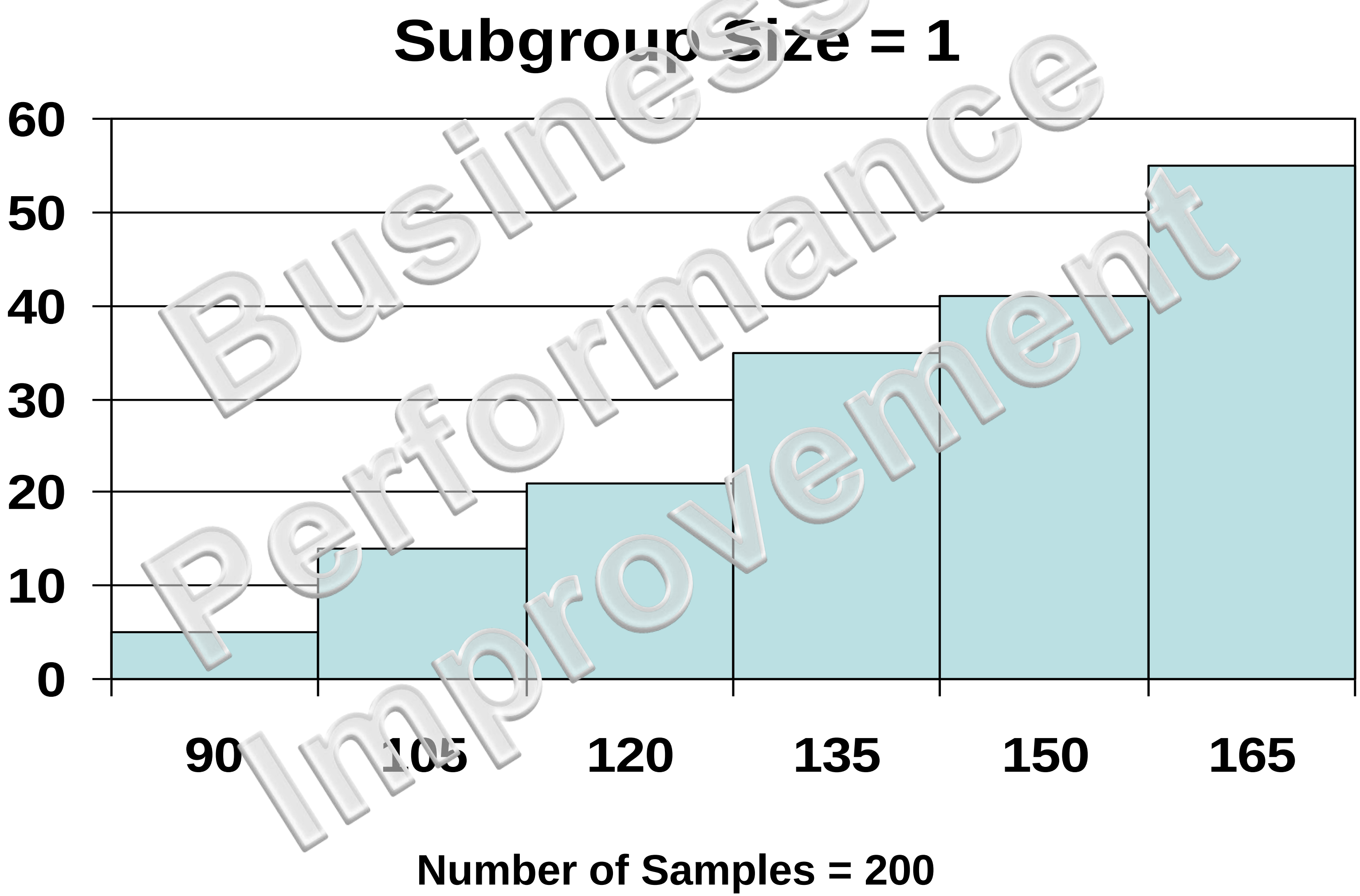
Central Limit Theorem



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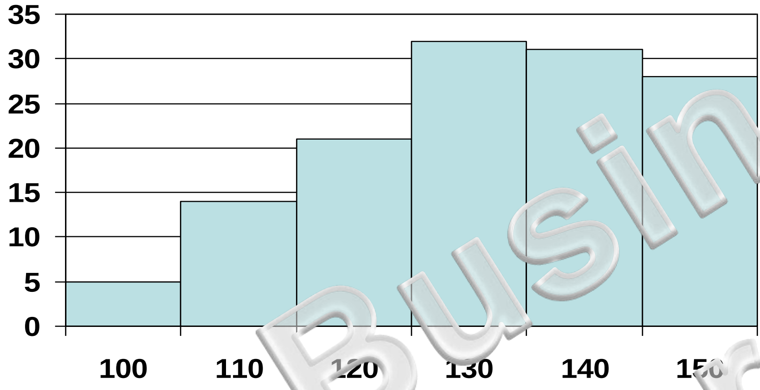
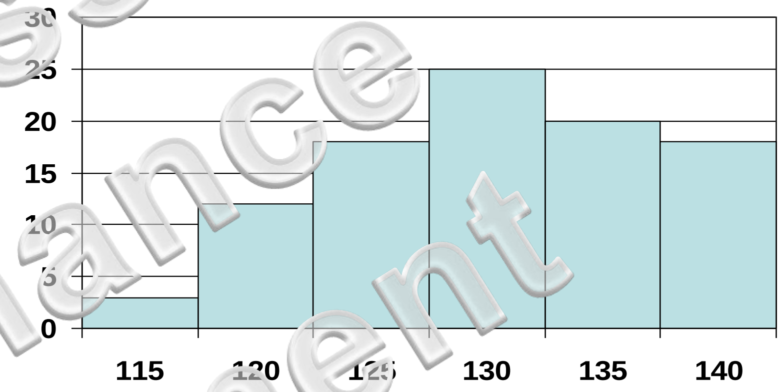
Central Limit Theorem

Subgroup Size = 2



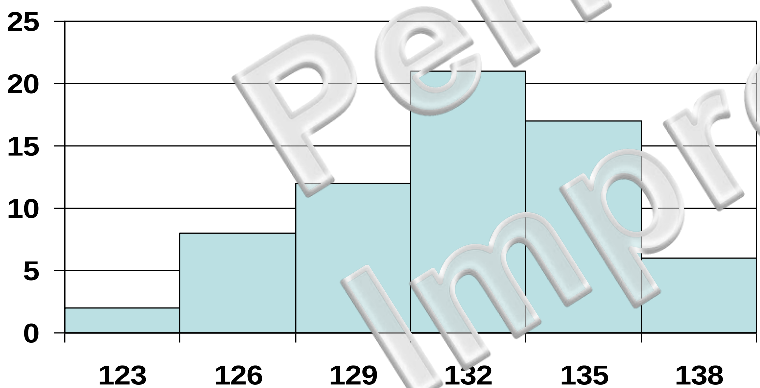
Number of Samples = 100

Subgroup Size = 3



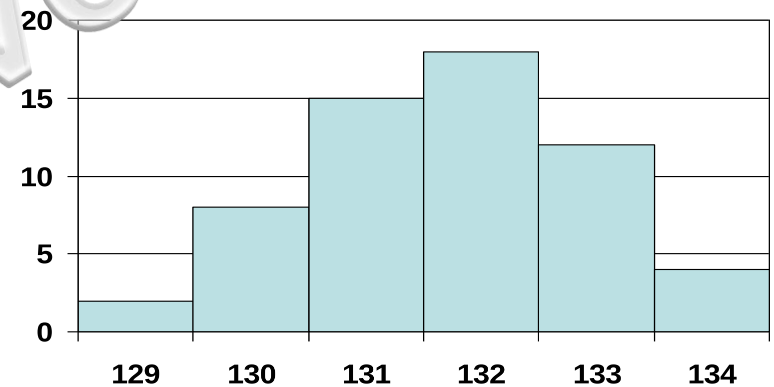
Number of Samples = 66

Subgroup Size = 4



Number of Samples = 50

Subgroup Size = 5



Number of Samples = 40

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Rational Subgroups

- In order to use Central Limit Theorem, must define the rational subgroups for data set
- Should be grouped in a way to:
  - maximize the chance of detecting shifts in process average (**between** subgroups)
  - minimize the variation (range) between samples within the subgroup (**within** subgroups)
- Don't subgroup different machine setups, different time of day, material differences, operators, or any other variables

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Central Limit Theorem Summary

- If you analyze the average of a subgroup, not the actual data values, it will be normally-distributed
- X-bar and R Chart applies Central Limit Theorem
  - Any data set can use the X-bar and R chart, regardless of data distribution (even non-normal)

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X-bar and R Charts

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Benefits of X-bar and R chart

- Can be used with any distribution of data
- Separates variation into two groups, for ease of investigation
  – Between subgroup (X-bar chart)
  – Within subgroup (Range chart)
- Quicker identification of out of control conditions than Individuals chart

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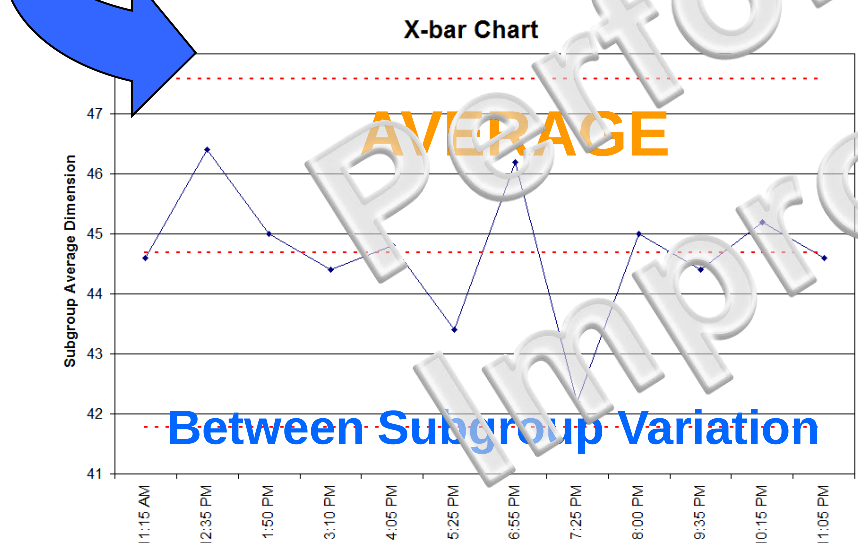
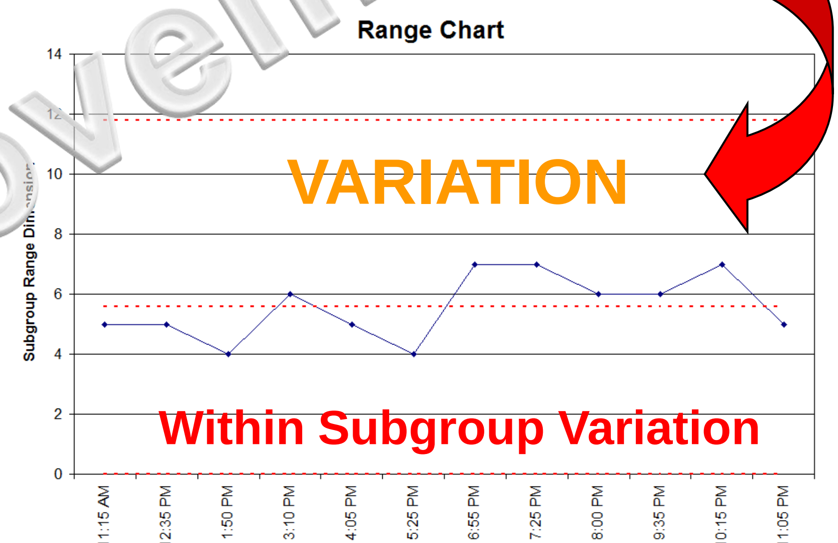
X-bar and R example

Subgroups	11:15 AM	12:35 PM	1:50 PM	3:10 PM	4:05 PM
SAMPLE 1	42	44	43	46	45
SAMPLE 2	43	46	46	47	43
SAMPLE 3	46	47	44	41	48
SAMPLE 4	47	46	47	43	45
SAMPLE 5	45	49	45	45	43
AVERAGE	44.6	46.4	45	44.4	44.8
MAX	47	49	47	47	48
MIN	42	44	43	41	43
RANGE (MAX-MIN)	5	5	4	6	5





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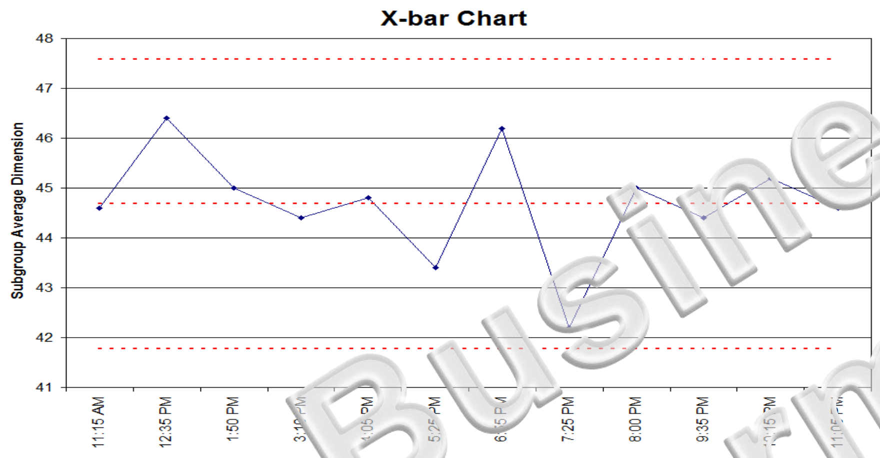
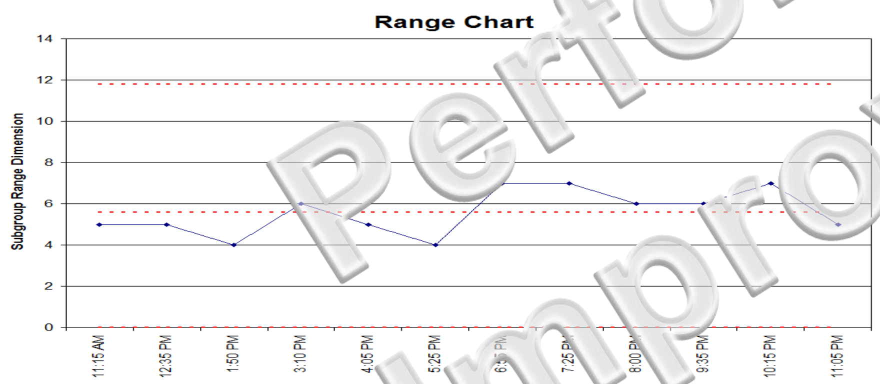
X-bar & R UCL and LCL



$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2 \overline{R}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2 \overline{R}$$



$$UCL_R = D_4 \overline{R}$$

$$LCL_R = D_3 \overline{R}$$

A_2, D_3 and D_4 are constants in table at end of section

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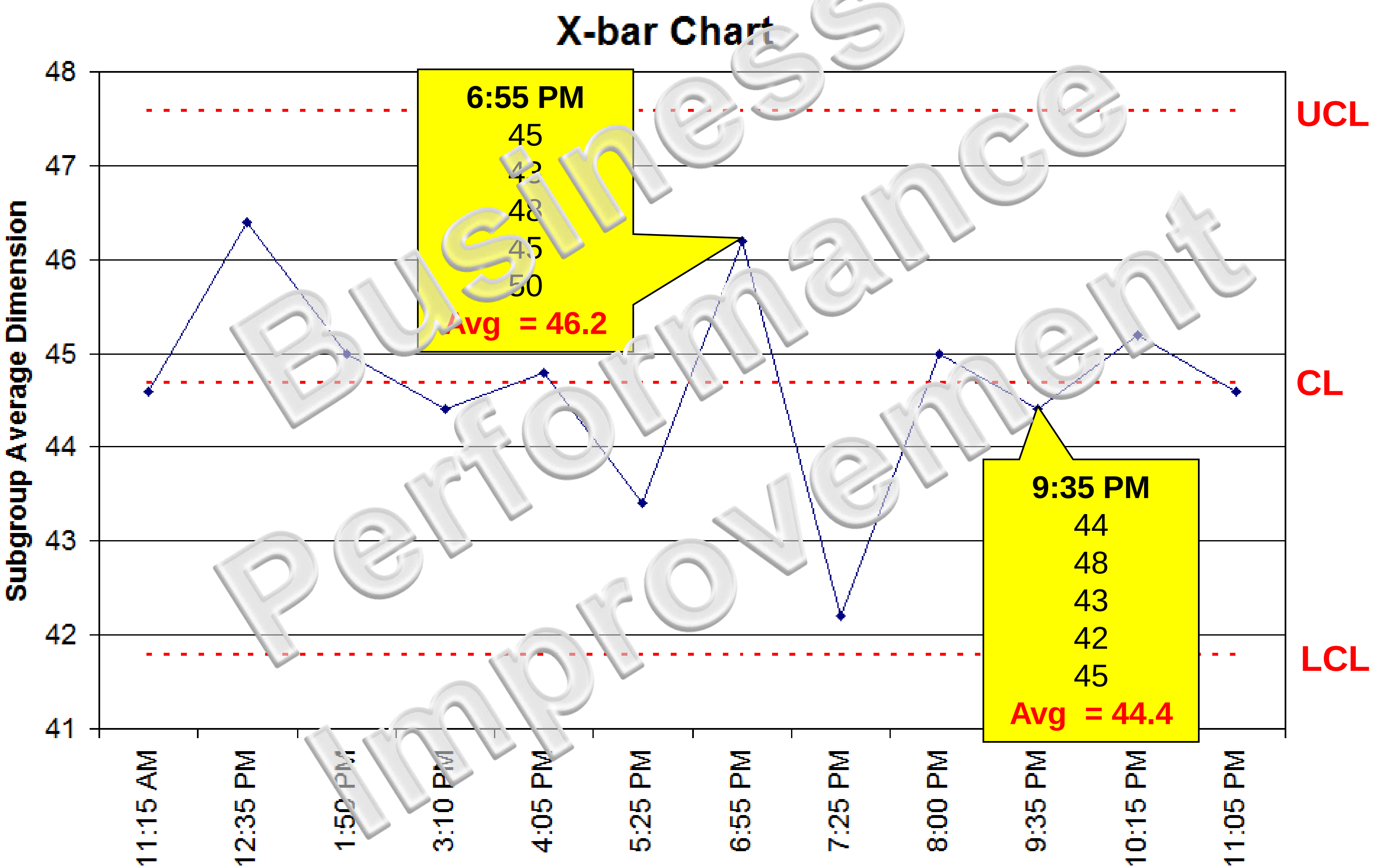
Average (X-bar) Chart



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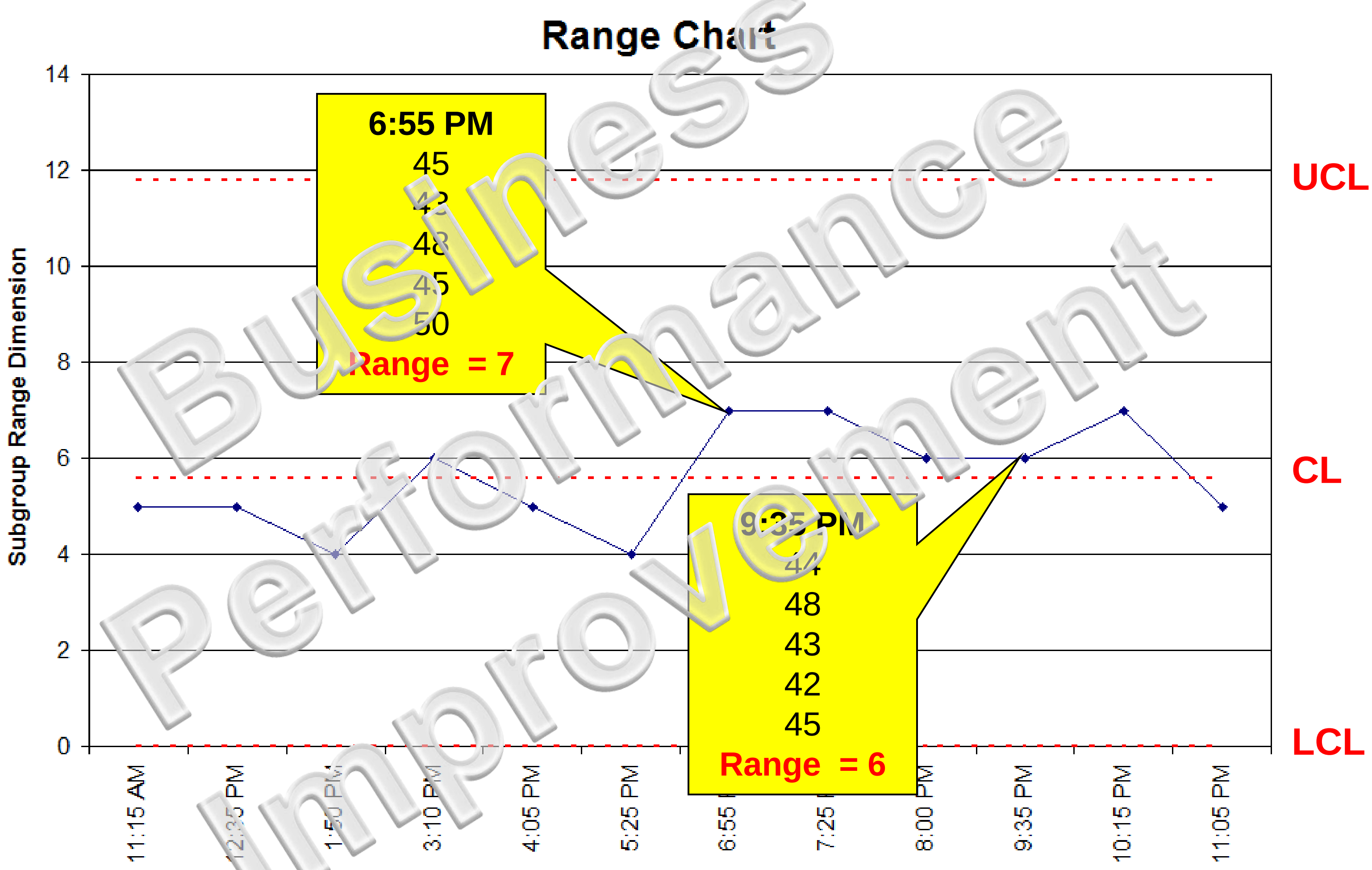
Range Chart



Range = Max of Data Subgroup – Min of Data Subgroup

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Normalized Charts

- If operating in a low volume situation, it is usually more practical to use one control chart for several parts or transactions
  - Data must be “normalized” before it is plotted to account for different spec limits of the data
- Process may have different limits or targets, want to know how close process is to target
  - Forecasting (Actual v. Predicted), product differences from same process, etc
  - Instead of plotting actual value, plot difference from nominal, target or expected

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Key Points for Normalized Charts

- Data should be from the same process
- Data should have similar variation
- Requires more data points to setup than traditional charts
- Uses same calculations as traditional charts, except use normalized values

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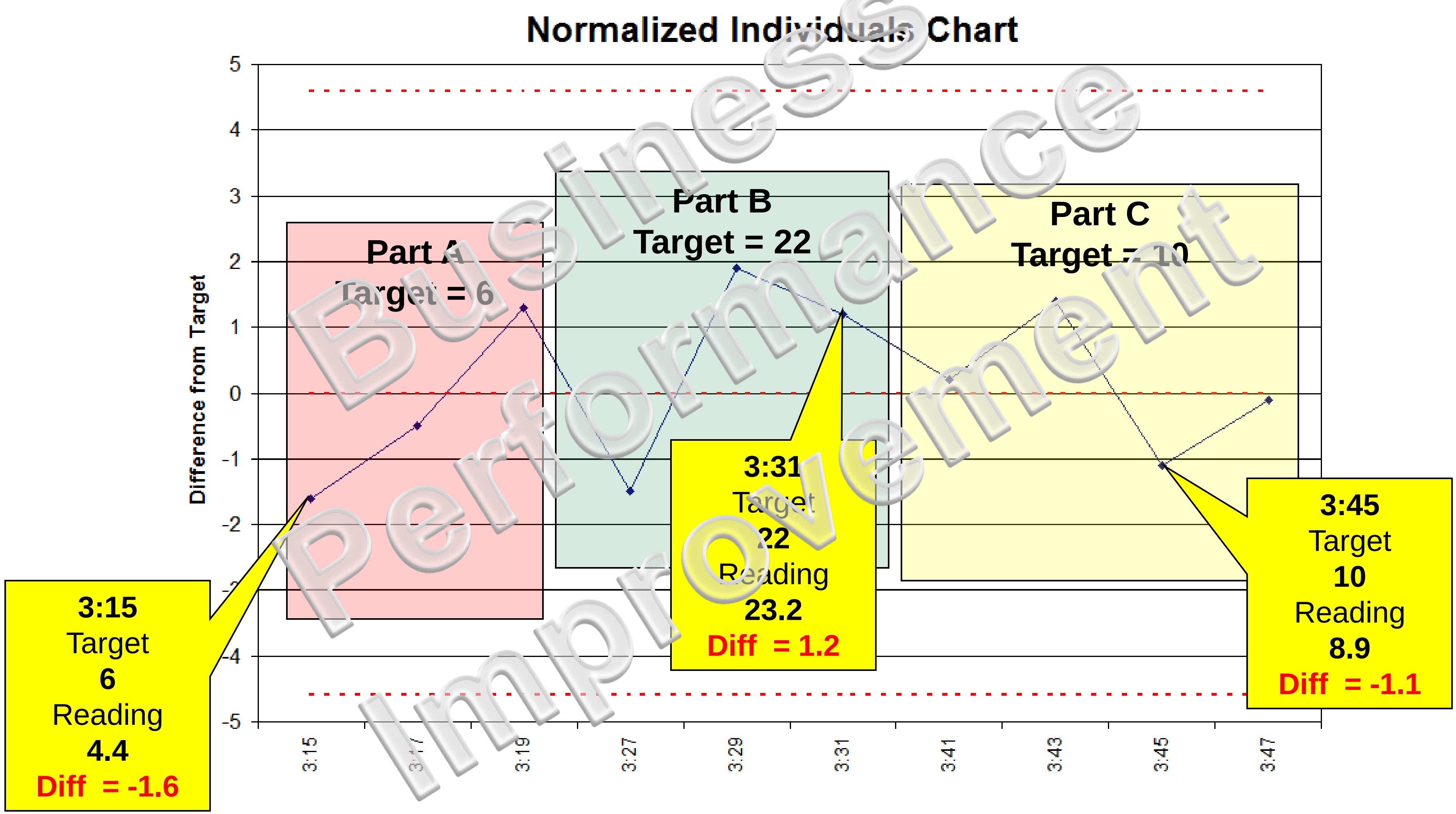
Example Normalized Chart



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Adjust Sampling

- As processes improve, reduce sampling frequency and/or subgroup size
  - Reduce as Cpk increases
  - More efficient use of resources
  - If process goes out of control, increase sampling frequency until process stabilized
- Concerns with reduced sampling
  - Less sensitive to small shifts in process
  - Can be harder to setup process for random sampling, than capturing all measurements

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Steps to Creating Control Charts

- Determine which process characteristic needs to be controlled
- Determine where in the process the control chart should be implemented
- Determine type of data needed to control process
- Choose the correct type of control chart
- Collect and calculate subgroup data
- Calculate centerlines and control limits
- Plot the data on the chart
- Interpret the chart for out of control conditions
- Improve process based upon analysis of control chart

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Advanced Control Charts

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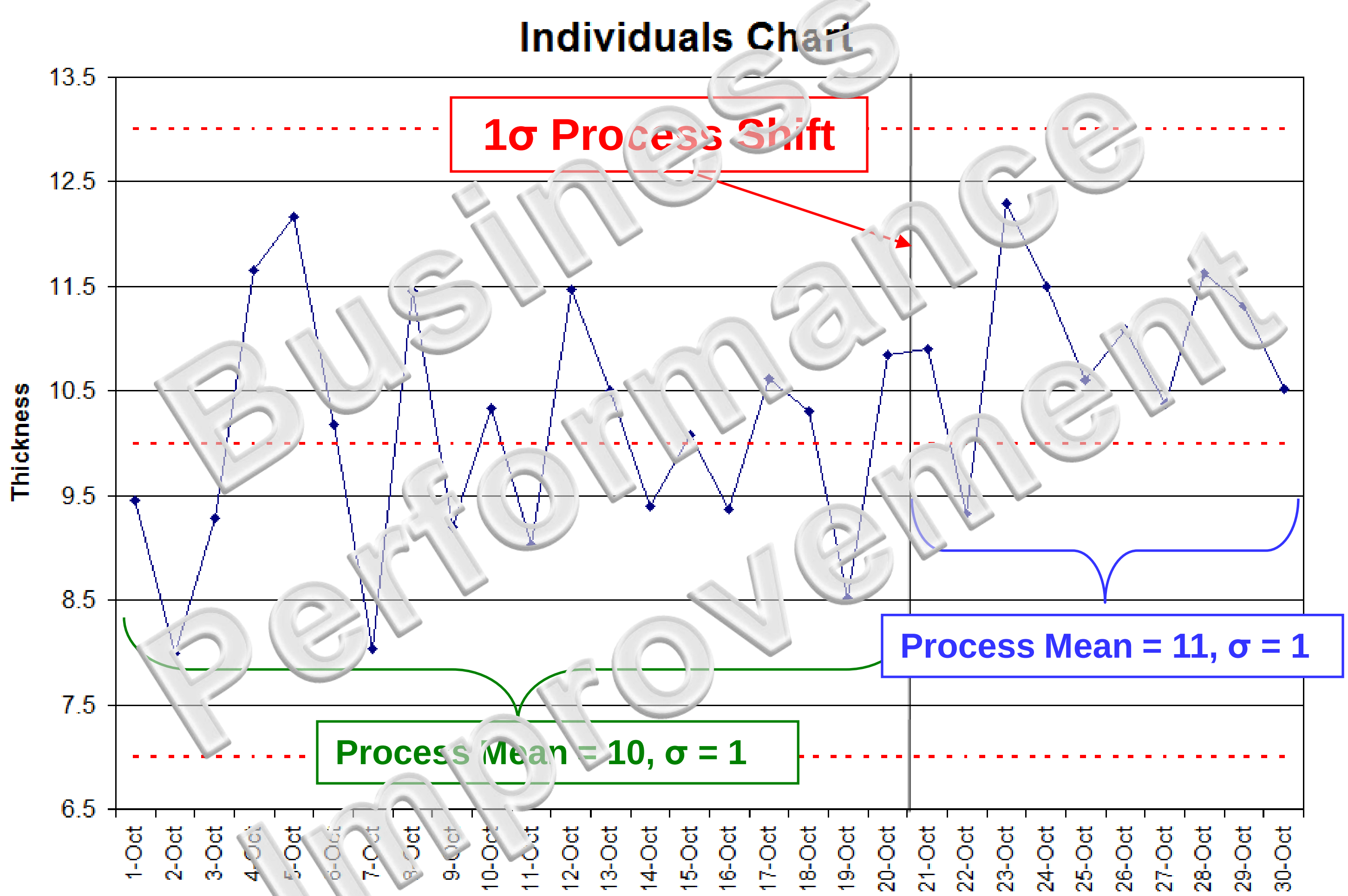
Small Shift Example



INDIVIDUALS CHART STILL SHOWS PROCESS IN CONTROL 10 DAYS AFTER SHIFT

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- Typical control charts don't use prior readings, only current data point
  - Make small shifts hard to detect (typically less than 1.5σ)
- Use prior readings in calculation of current data point to detect small shifts
- Two alternative charts are CUSUM and EWMA

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- **Cu**mulative **Sum**
- Data converted to difference from mean, and added to previous value

Difference from Mean (μ = 10)

Date	Value	Value-mean	New Value
1-Oct	9.45	-0.55	**-0.55**
2-Oct	7.99	**-2.01**	-2.56
3-Oct	9.29	-0.71	**-3.27**
4-Oct	11.66	**1.66**	-1.61

-2.56 = -2.01 + -0.55

-1.61 = 1.66 + -3.27

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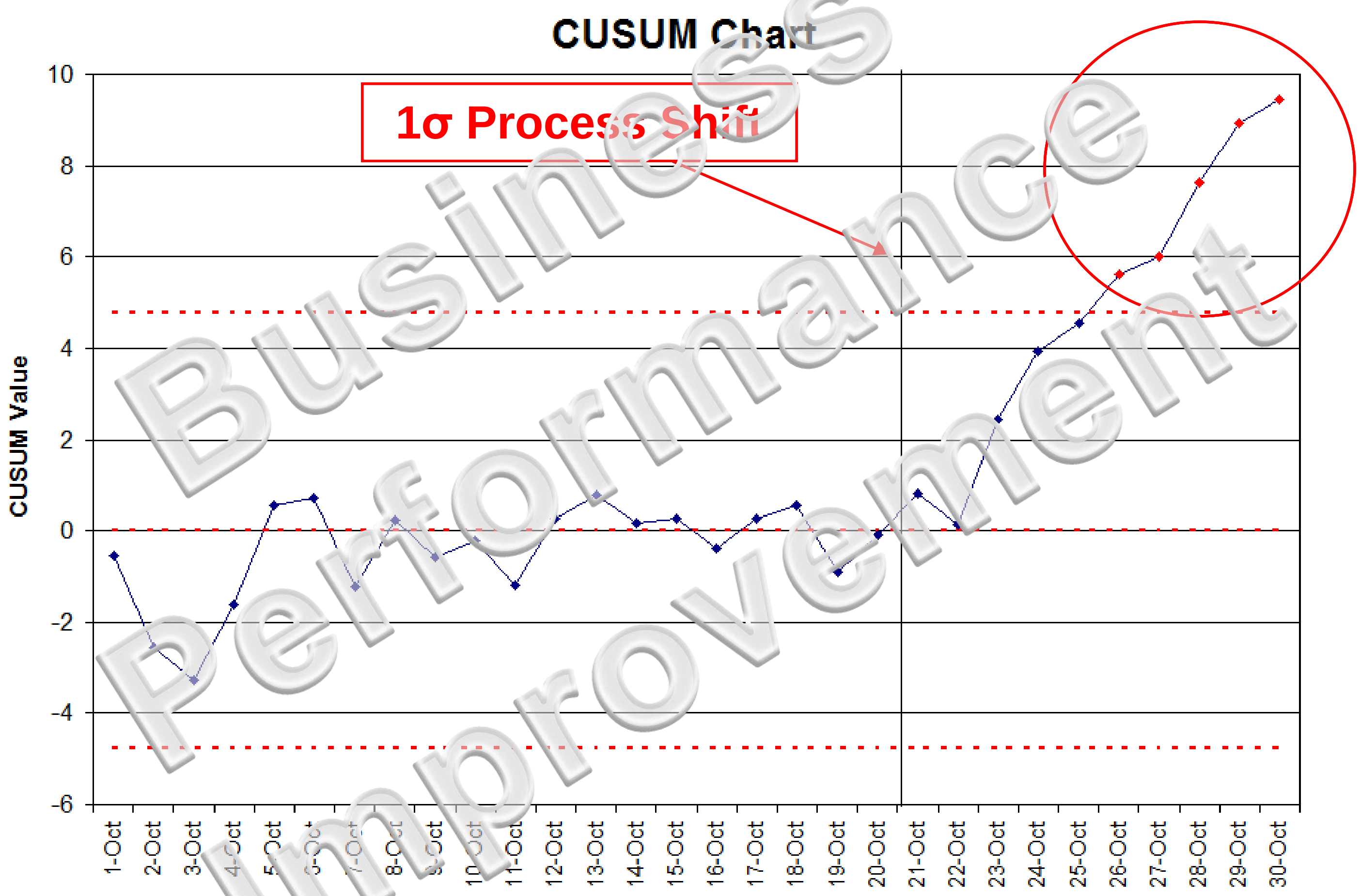
CUSUM Chart detects shift earlier!



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CUSUM CHART DETECTS SMALL SHIFT ONLY 6 DAYS AFTER SHIFT (4 DAYS EARLIER)

How to set control limits for CUSUM

Table for obtaining CUSUM limits similar to Shewhart control charts

k	0.25	0.5	0.75	1.0	1.25	1.5
Mean Shift	0.5	1.0	1.5	2.0	2.5	3.0
Decision (h)	8.01	4.77	3.34	2.52	1.99	1.61

Example: To detect a mean shift of 1.5 standard deviations, you would set k = 0.75, and the limits on the CUSUM chart would be computed using h*σ (3.34σ), which would be 3.34(1) = ± 3.34

Prior example used shift of 1σ, k=0.5 and h=4.77, so limits were set at ±4.77

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- **E**xponentially **W**eighted **M**oving **A**verage
- Good for detecting small shifts in processes
- Better at detecting large shifts than CUSUM
- Uses prior readings in calculation of current data point
  - the further away, the less influence
- Best if used with data from individuals chart (subgroup size = 1)

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How to setup EWMA chart

- Determine λ (between 0 and 1)
  - λ is the proportion of current value used for calculating newest value
  - Recommend λ = 0.10, 0.20 or 0.40 (use smaller λ values to detect smaller shifts)
- Calculate new z values using λ = 0.10

$$z_i = \lambda * x_i + (1 - \lambda) * z_{i-1}$$ (where i = sample number)

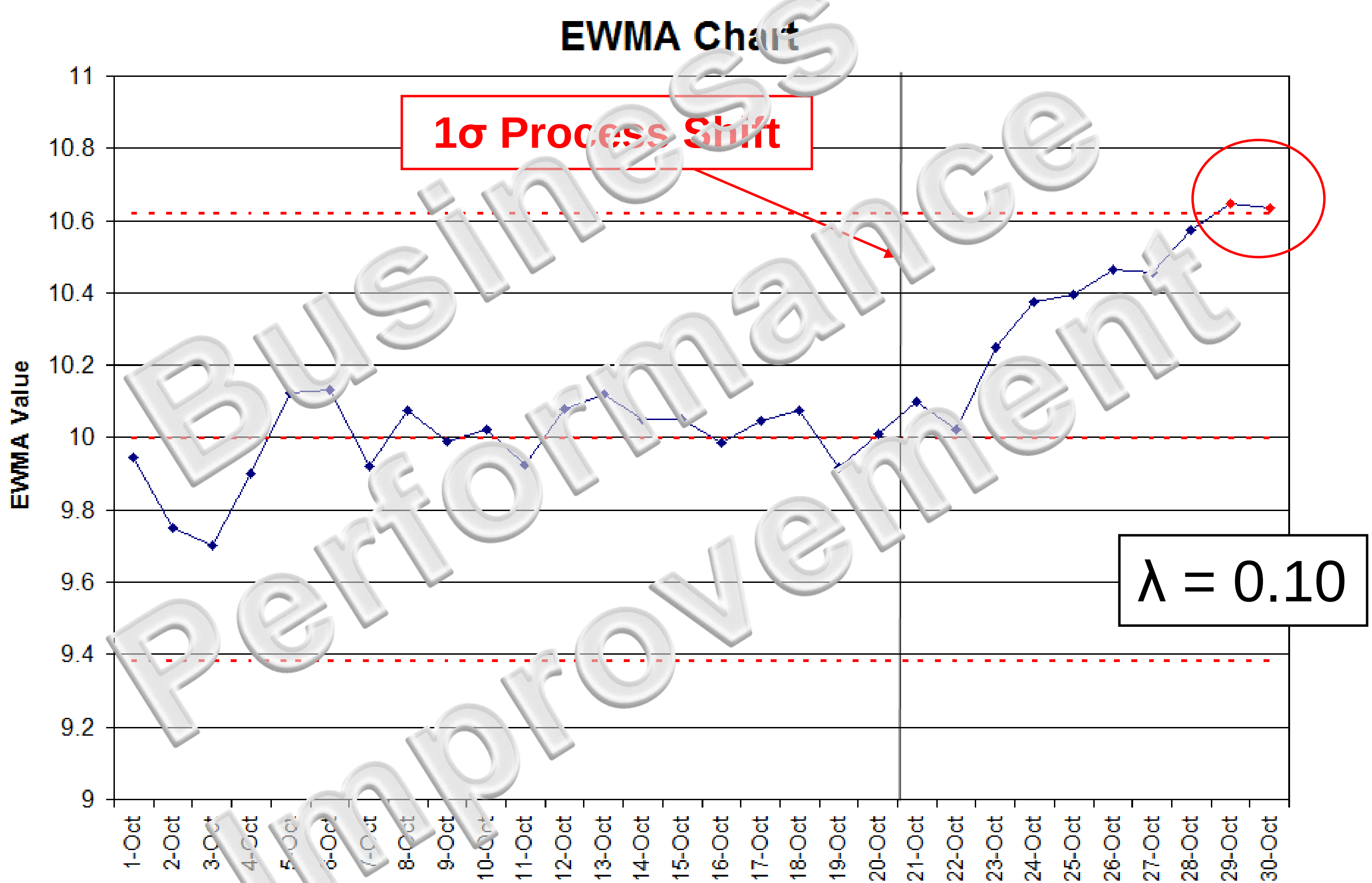
Sample	Date	x	z
1	1-Oct	9.45	**9.945**
2	2-Oct	**7.99**	9.7495
3	3-Oct	9.29	**9.7035**
4	4-Oct	**11.66**	9.899

9.7495 = (0.9*9.945) + (0.1*7.99)

9.899 = (0.9*9.7035) + (0.1*11.6)

Business Performance Improvement

EWMA Chart detects shift earlier!



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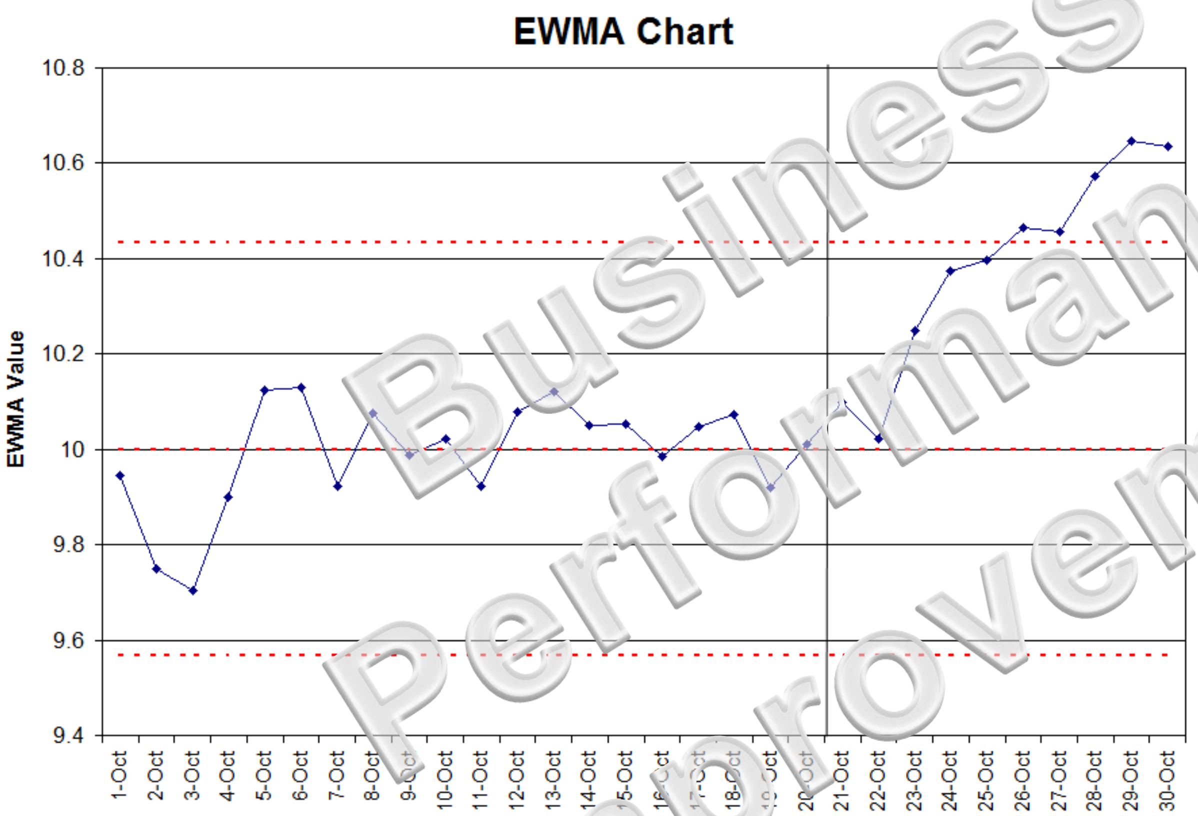
EWMA CHART DETECTS SMALL SHIFT 2 DAYS EARLIER THAN INDIVIDUALS CHART

EWMA UCL and LCL



$$UCL_z = \overline{u} + L\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL_z = \overline{u} - L\sigma\sqrt{\frac{\lambda}{2-\lambda}}$$

EWMA Chart uses L for determining limits, typically set to 2.7

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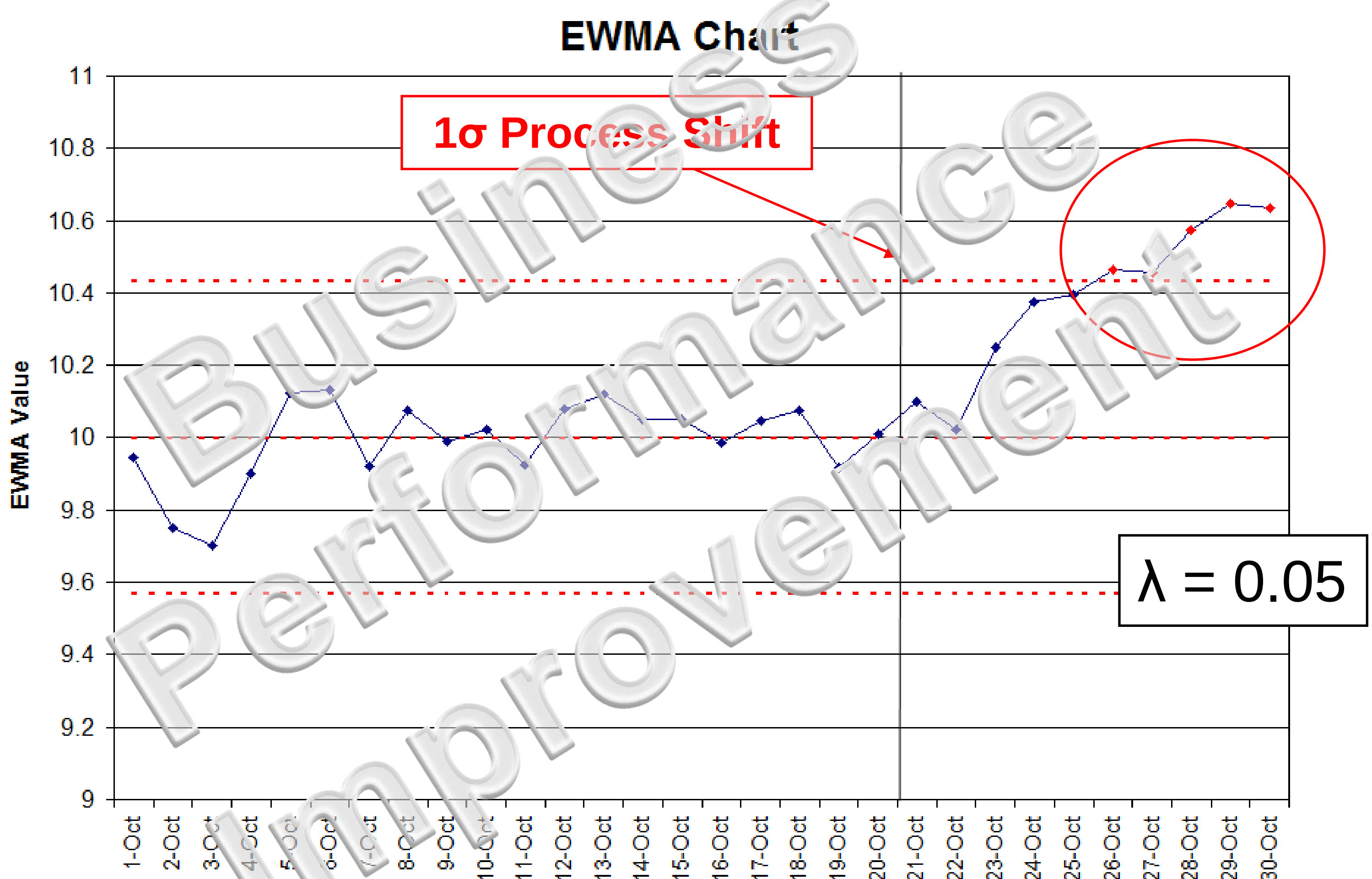
EWMA Chart detects shift even earlier!



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EWMA CHART NOW DETECTS SMALL SHIFT SIMILAR TO CUSUM CHART

Summary of CUSUM and EWMA

- **CUSUM**
  - Uses current reading difference from mean, and previous CUSUM value, equally weighted
- **EWMA**
  - Uses weighted current reading and weighted previous EWMA value
  - Weight of each value is determined by user, based on needs of chart

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Disadvantages to CUSUM and EWMA

- Cannot quickly detect large shifts in process, like traditional control charts
- Use CUSUM and EWMA for detecting small shifts, Individuals and X-bar charts for larger shifts
  - Ideally, use both within the process, to detect both small and large shifts, with limits set to $\pm 3.5\sigma$

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Attribute Control Charts

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Attribute Control Charts

- p chart
- np chart
- c chart
- u chart

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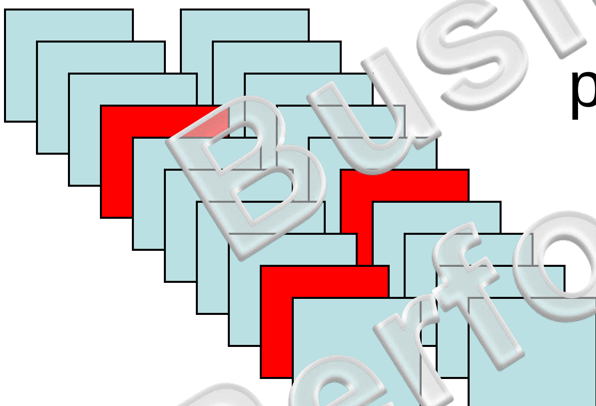
- Plots the percentage of defectives within a sample



3 red defective parts out of 20 parts

p = 3 / 20 = **15%** defect rate

- Use when sample size varies
  - – Control limits adjust according to sample size

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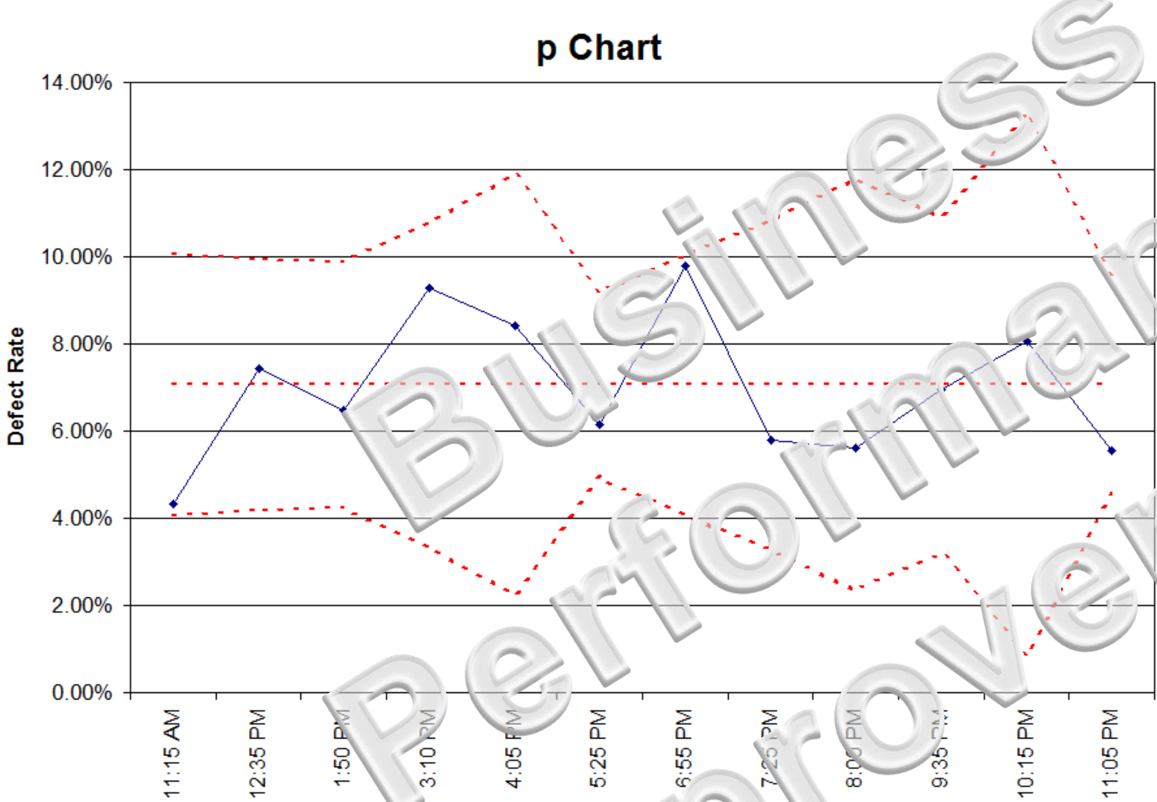
p chart UCL and LCL



$$UCL_p = \overline{p} + 3\sqrt{\frac{\overline{p}\,(1 - \overline{p})}{n}}$$

$$LCL_p = \overline{p} - 3\sqrt{\frac{\overline{p}\,(1 - \overline{p})}{n}}$$

p Chart has adjusting control limits, based on sample size

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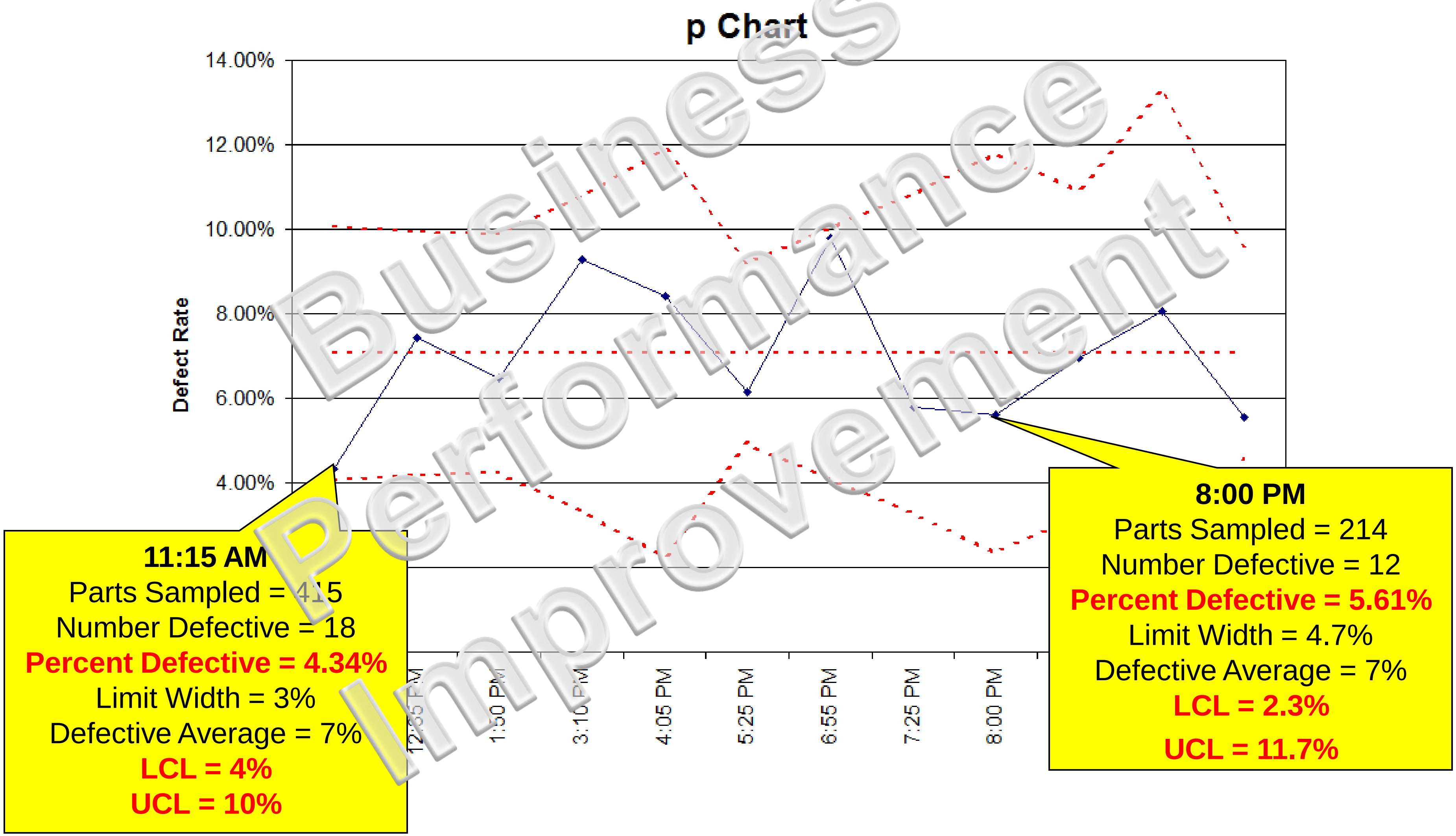
p chart



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np chart

- Plots the number of defective parts in a sample
- Requires the same sample size each time
- Easy to use, since no calculations required

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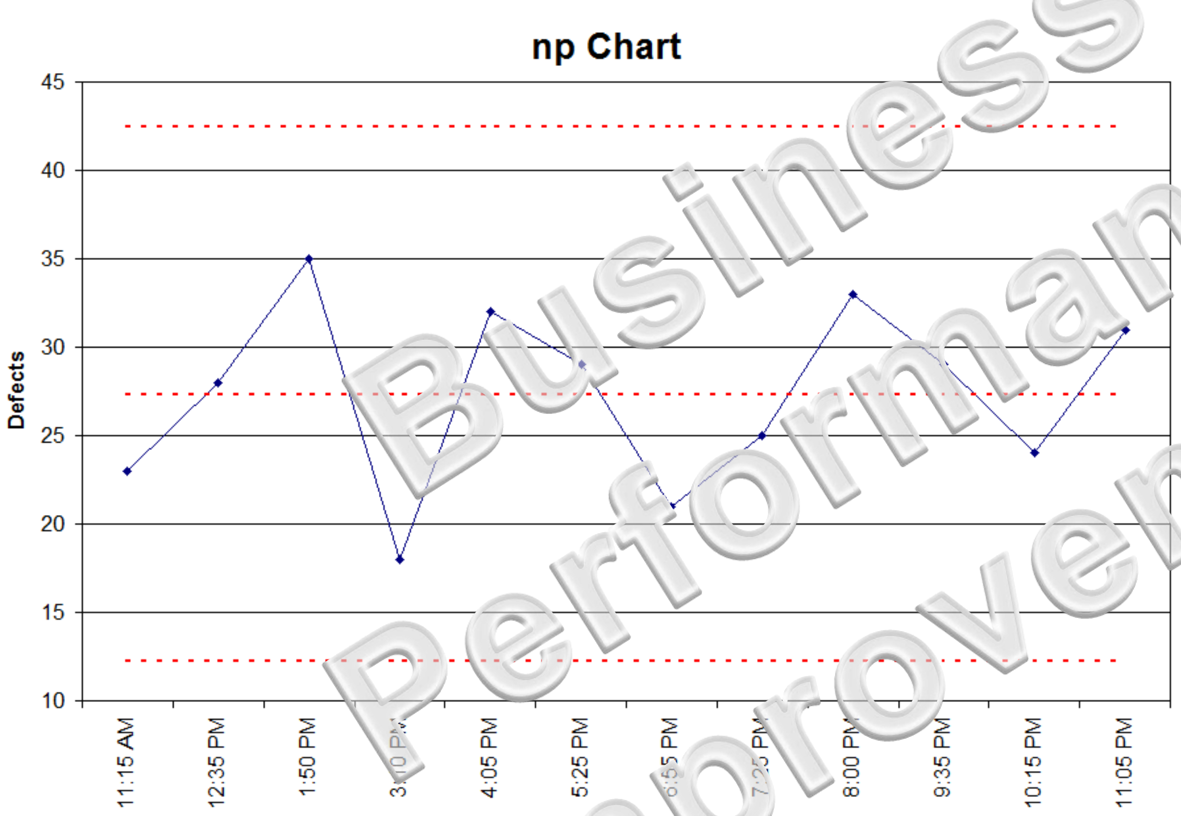
np chart UCL and LCL



$$UCL_{np} = n\overline{p} + 3\sqrt{n\overline{p}\,(1 - \overline{p})}$$

$$LCL_{np} = n\overline{p} - 3\sqrt{n\overline{p}\,(1 - \overline{p})}$$

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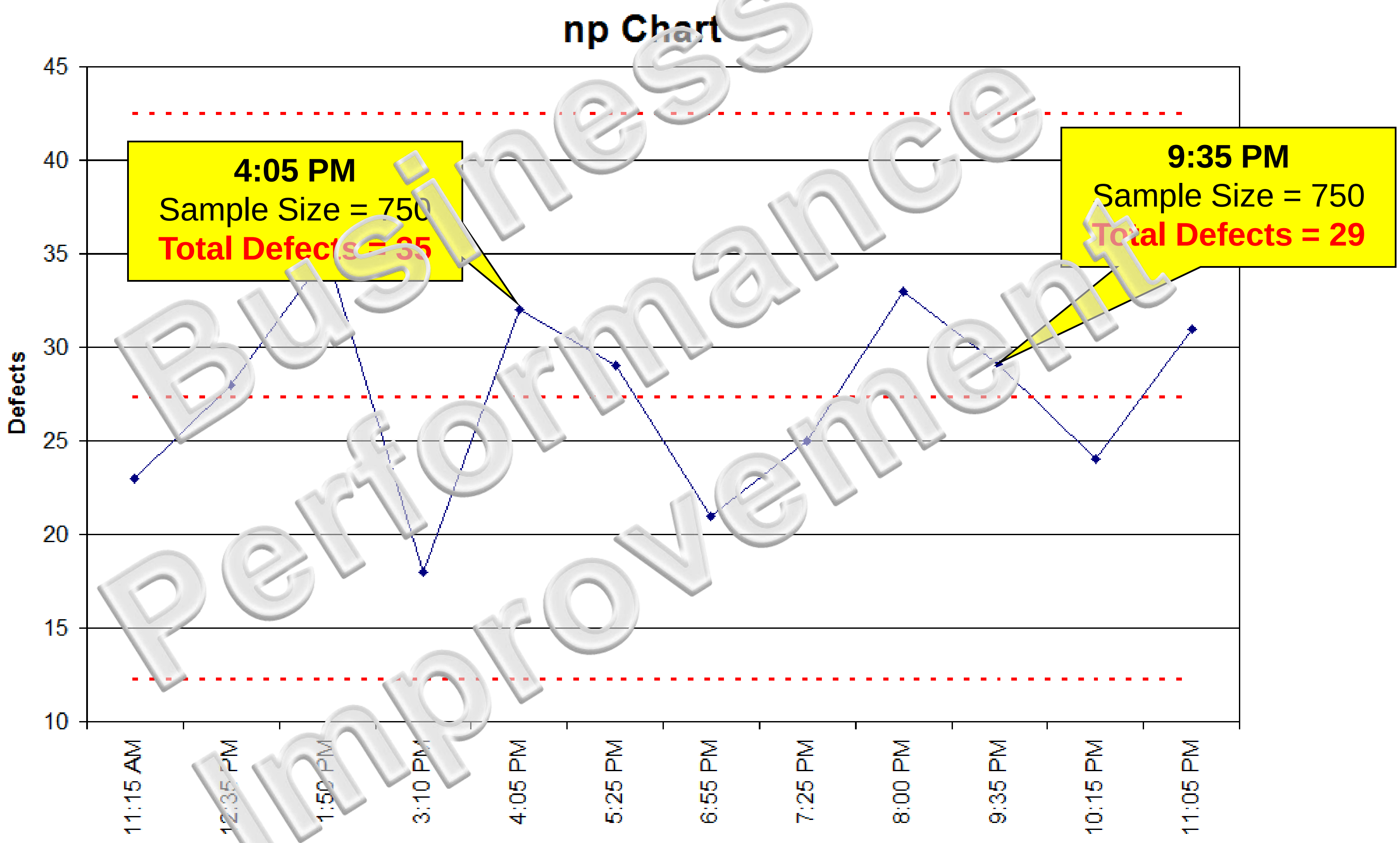

This specific chart uses 750 samples each time

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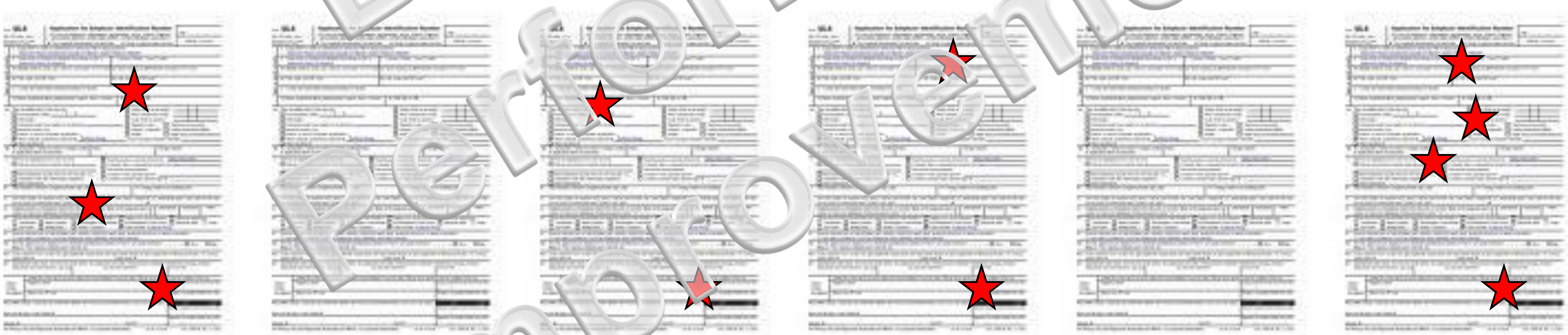
- Plots the quantity of defects per part in a sample
- Each part can have more than one defect
- Use when sample size varies



11 total defects found on 6 documents

u = 11/6 = **1.833** defects per document

whereas p = 4/6 = **67%** defect rate

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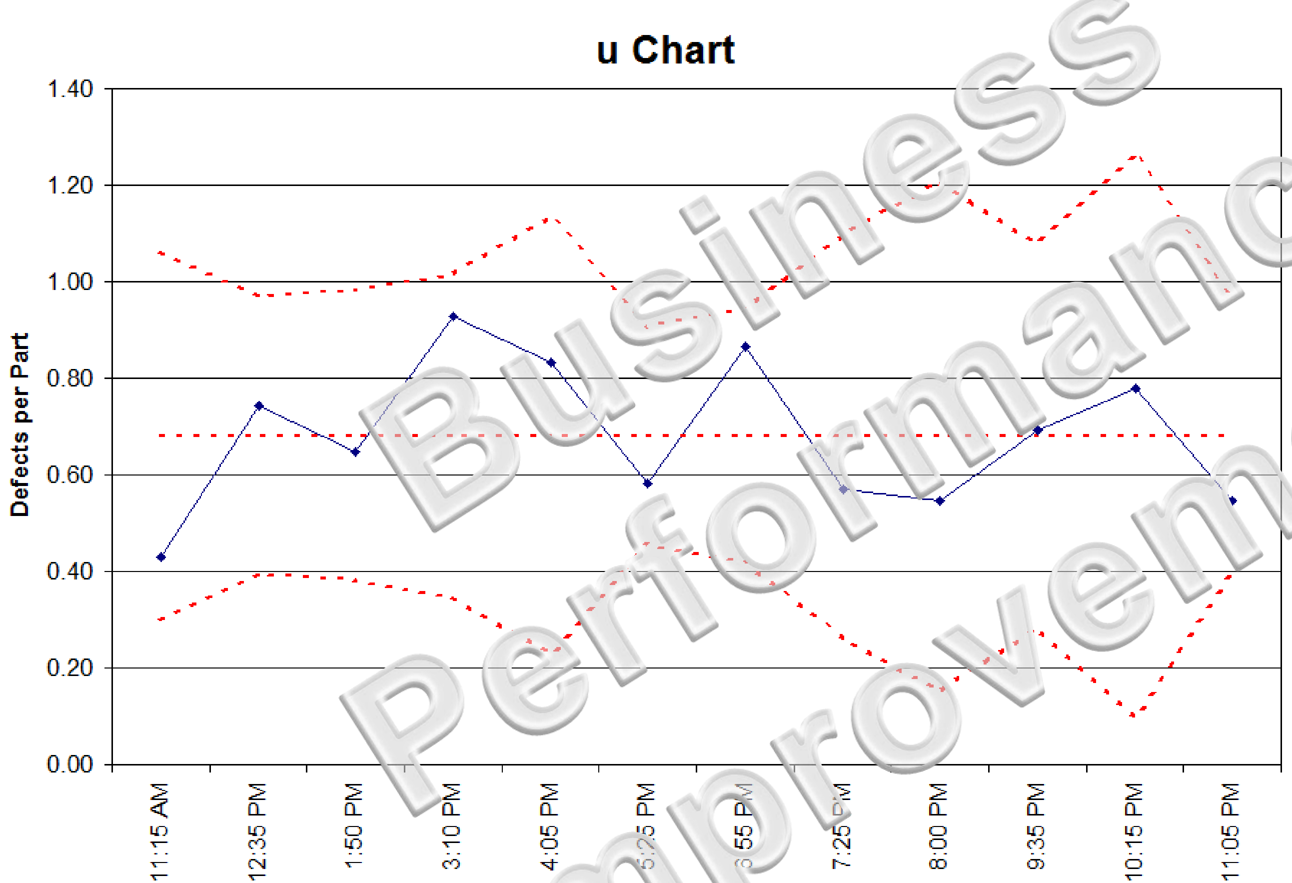
u chart UCL and LCL



$$UCL_u = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}}$$

$$LCL_u = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}}$$

u Chart has adjusting control limits, based on sample size

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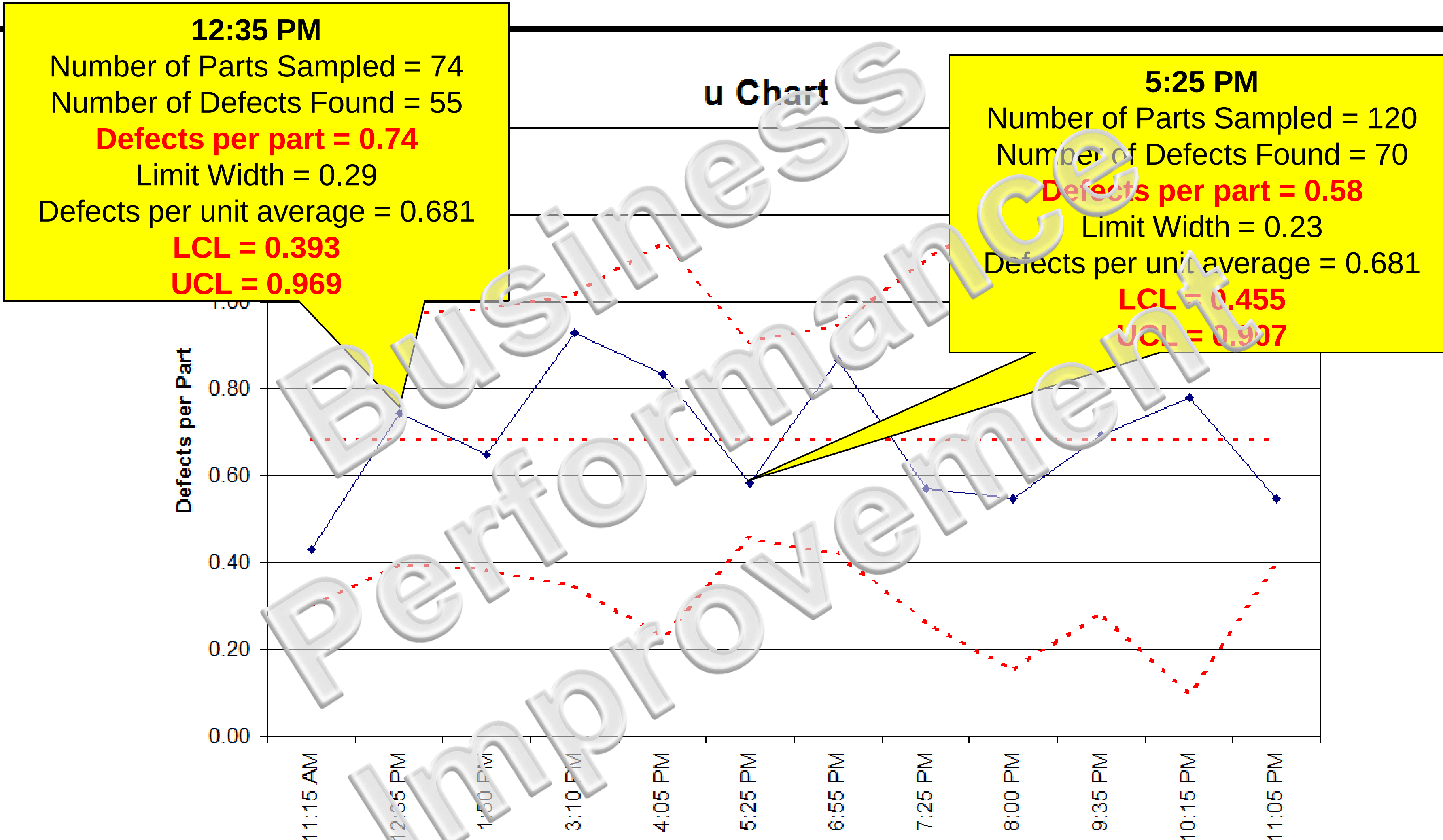
u Chart



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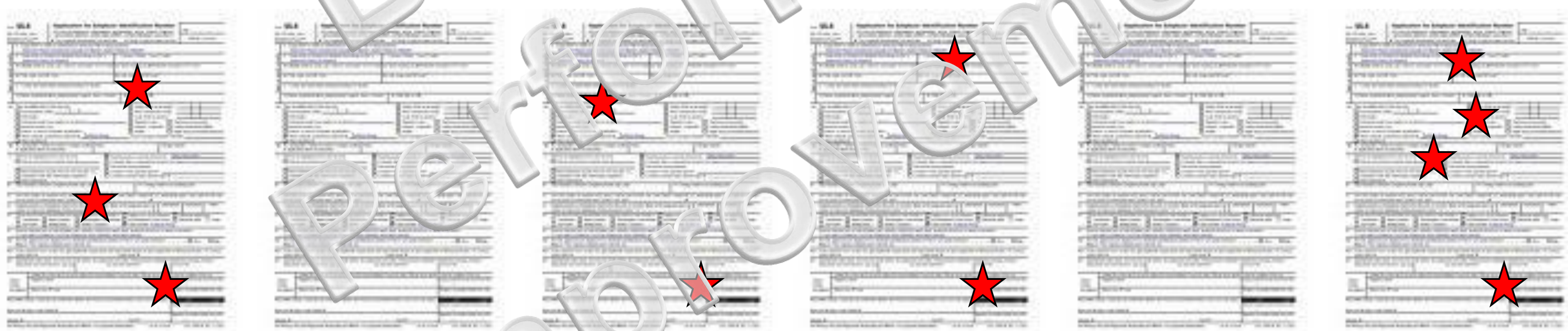
c chart

- Plots the quantity of defects in a sample
- Each part can have more than one defect
- Requires same number of parts within each sample



11 total defects found on 6 documents

c = **11** defects per sample

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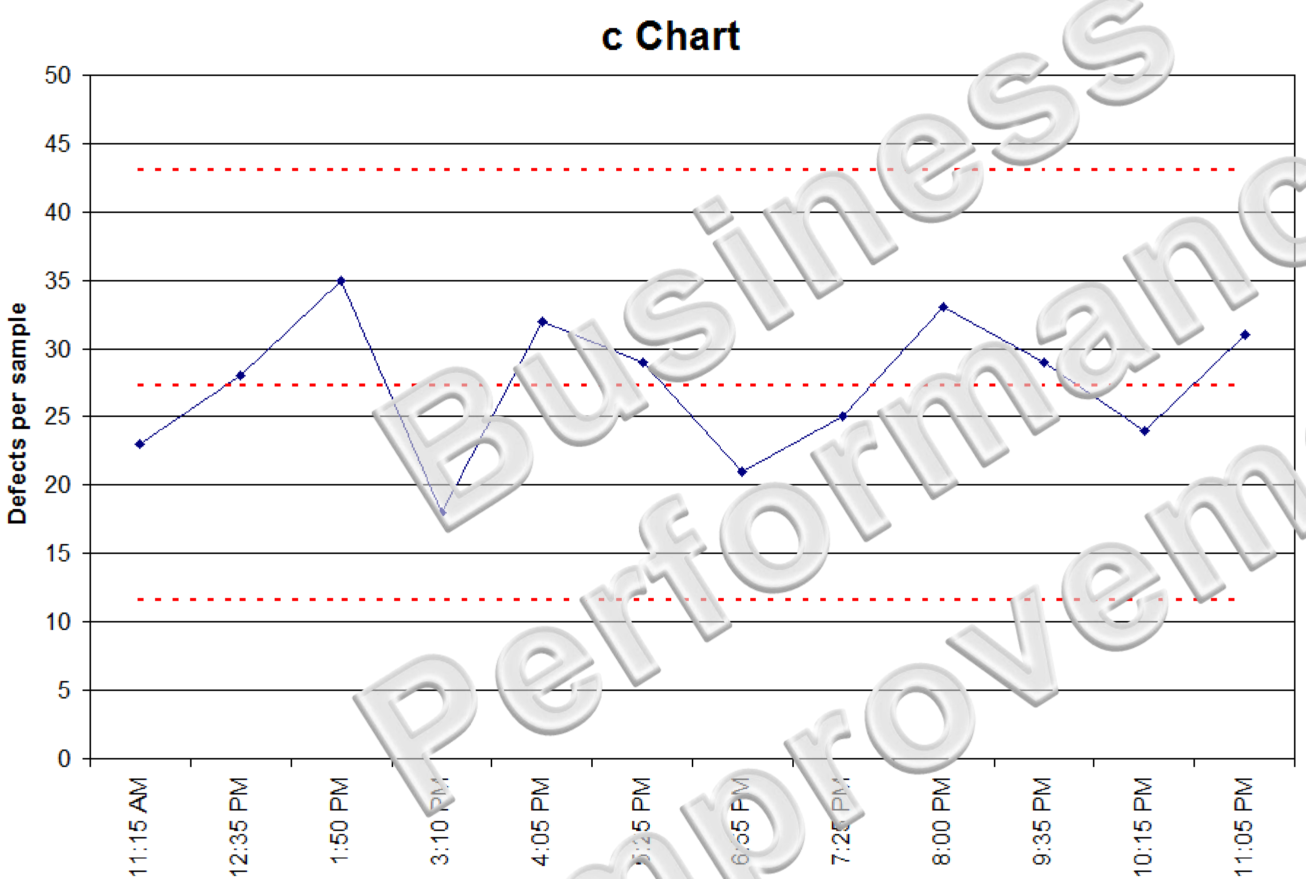
c chart UCL and LCL



$$UCL_c = \overline{c} + 3\sqrt{\overline{c}}$$

$$LCL_c = \overline{c} - 3\sqrt{\overline{c}}$$

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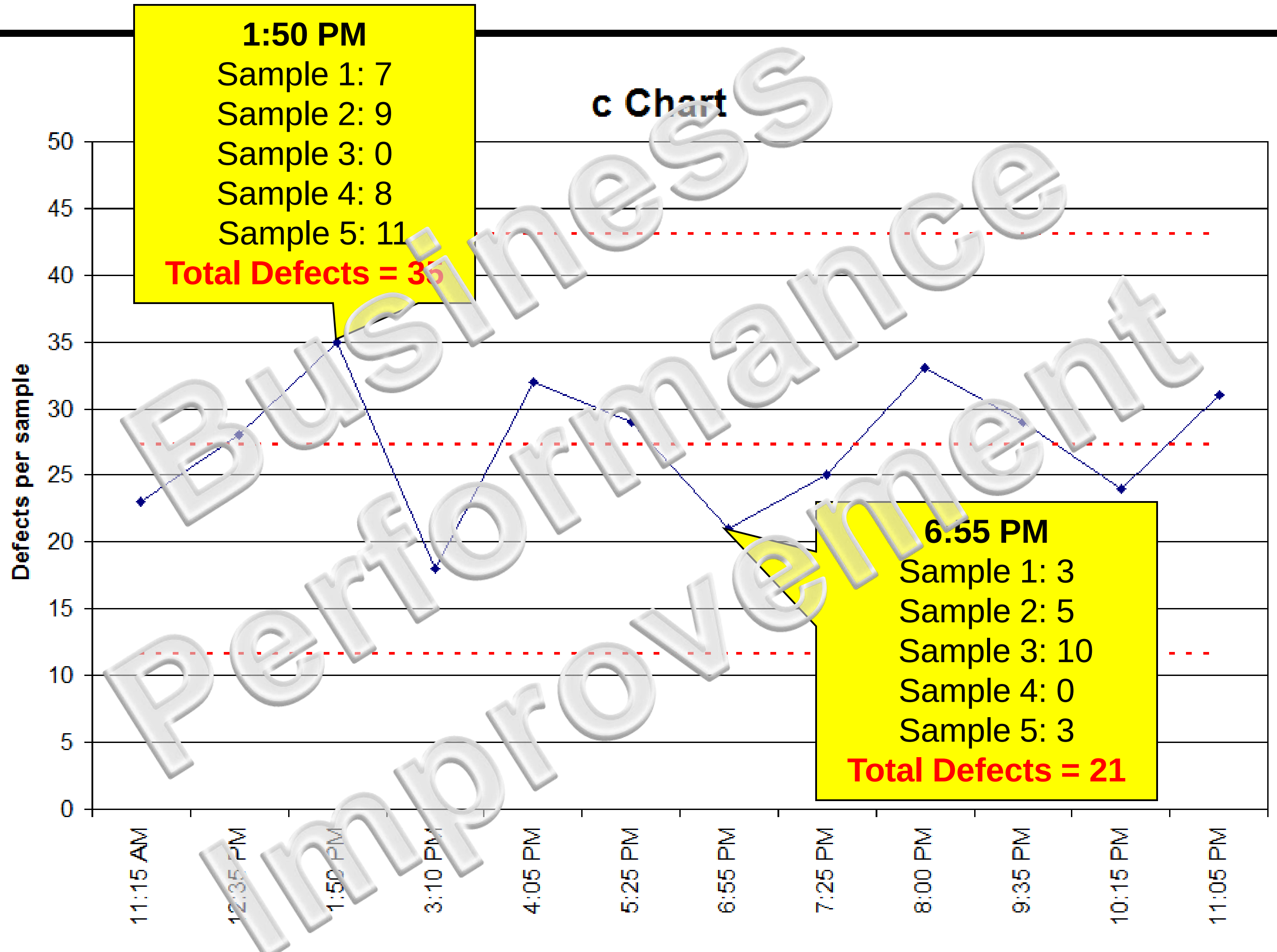
c Chart



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This specific chart uses 5 samples each time

Attribute Selection Chart

Data Type

Sample Size

Attribute Charts	Defectives (Pass/Fail)	Count of Defects
Constant	np	c
Varies	p	u

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Final Points

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Control Chart Implementation Phases

- **Phase 1**: Control Charts on few process outputs (key quality characteristics)
- **Phase 2**: Expansion of charts, including numerous attribute control charts
- **Phase 3**: Some control charts on few process inputs (key process parameters), some converted to X-bar and R charts
- **Phase 4**: Attribute control charts replaced with variable charts, many charts deleted due to non-criticality or ineffectiveness
- **Phase 5**: Most charts are X-bar and R charts on key process parameters, some advanced charts (CUSUM, EWMA, etc)

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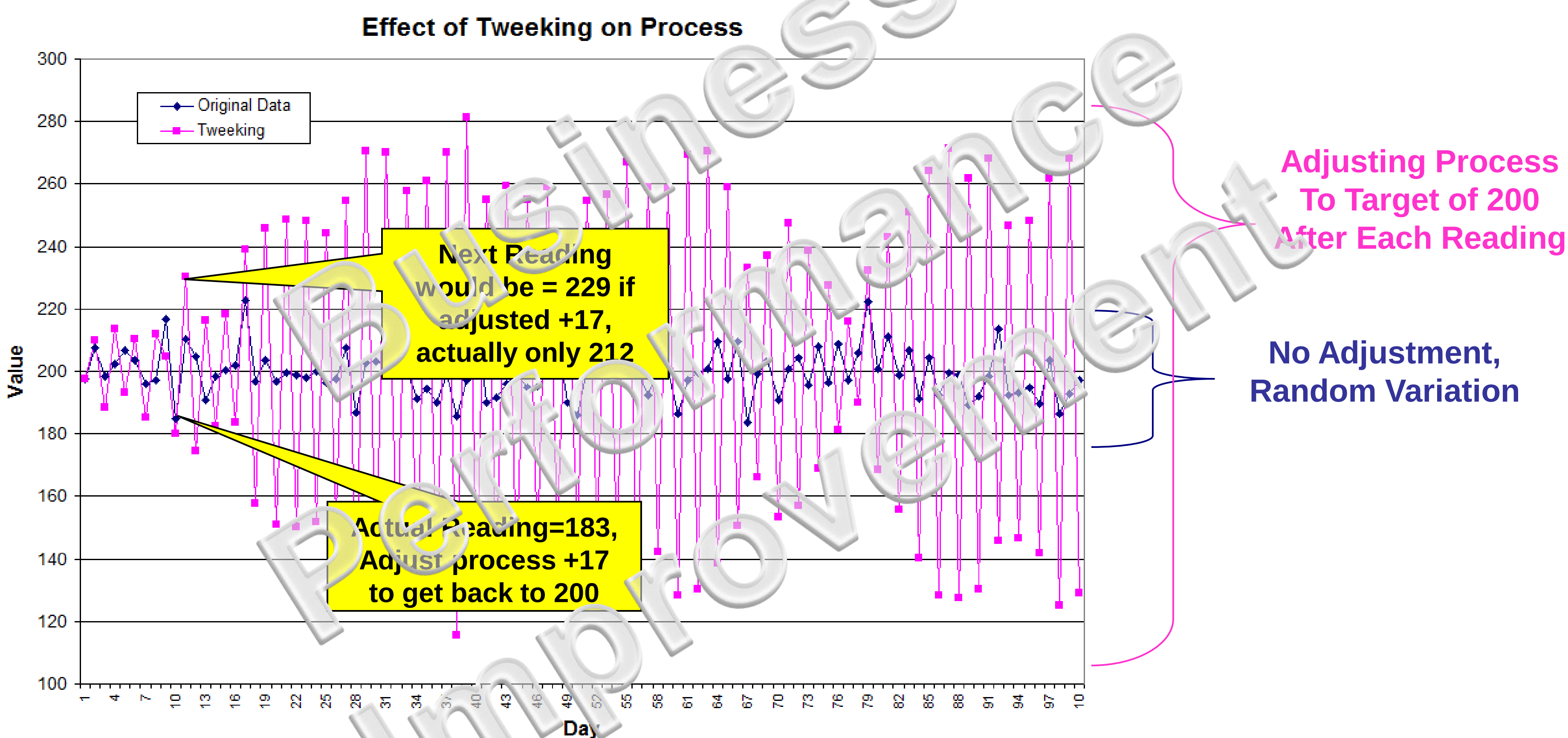
What if you react to common cause?



Adjusting process back to center will increase overall variation

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Other Key Points

- Create a control chart and histogram before performing any data analysis or calculations
- Use at least 20 data points before calculating control limits
- Moving Range charts are optional, and not required with Individuals charts
- Spec limits may be displayed on Individuals charts, but can lead to complacency, never on X-bar and R charts
- Apply SPC to the inputs of a process whenever possible
- Use X-bar and Standard Deviation (S) chart instead of X-bar and R chart when subgroup size > 5
- Special causes are indications of potential problems, they cannot guarantee that a problem exists
- Document any out of control condition observed on a chart, to show evidence that it was seen, and some investigation took place

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Common Pitfalls of SPC Failure

- Lack of commitment from management
  – Resistance to change from reactive to preventative
- Lack of training and education in SPC
  – Misinterpretation of control charts
  – Lack of focus and/or maintenance on control charts
- Inadequate measurement system in place (poor Gage R&R)

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Reference Section

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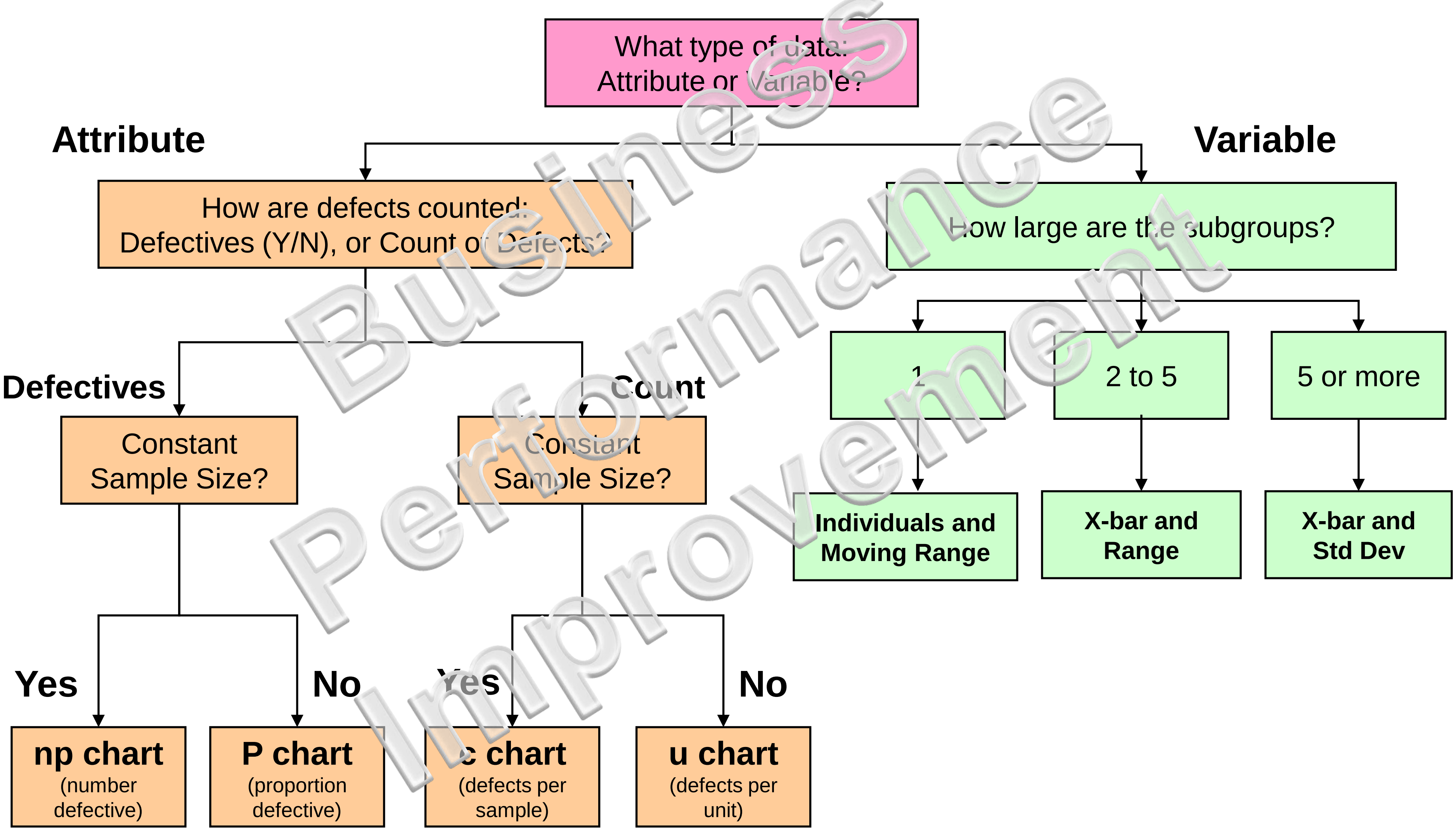
Decision Tree for Control Charts



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Shewhart Control Chart Constants

n	E_2	D_4	D_3	A_2
2	2.66	3.27	0.00	1.88
3	1.77	2.57	0.00	1.02
4	1.46	2.28	0.00	0.73
5	1.29	2.11	0.00	0.58
6	1.18	2.00	0.00	0.48
7	1.11	1.92	0.08	0.42
8	1.05	1.86	0.14	0.37
9	1.01	1.82	0.18	0.34

n is the subgroup size

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Additional Resources

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http://www.biz-pi.com

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